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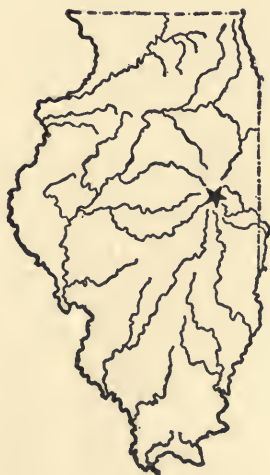
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UNIVERSITY OF ILLINOIS
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GROWTH AND SENESCENCE IN
PUREBRED JERSEY COWS

By F. A. DAVIDSON



URBANA, ILLINOIS, JANUARY, 1928

CONTENTS

	PAGE
INTRODUCTION.....	183
THE PROBLEM.....	184
SOURCE OF DATA.....	185
BIOMETRICAL ANALYSIS OF DATA.....	185
Course of Growth in Register-of-Merit Jersey Cows as Described by In- crease in Body Weight with Advancing Age.....	185
Truncation of Yearly Butterfat Frequency Distributions Due to Selective Effect of Register-of-Merit Requirement.....	199
Comparison of Means (\bar{V}) and Standard Deviations (σ) on Original Scale with Means (a) and Standard Deviations (s) on Logarithmic Scale.	209
Course of Growth and Senescence as Described by Rise and Fall in Yearly Butterfat Yields with Advancing Age.....	212
Influence of Level of Production Upon Age Curve of Milk Secretion.....	223
Nature of Selection of Register-of-Merit Production Requirement.....	224
SUMMARY.....	225
Course of Growth in Body Weight.....	226
Course of Growth and Senescence as Described by Rise and Fall in Yearly Butterfat Yields with Advancing Age.....	226
LITERATURE CITED.....	229
APPENDIX.....	231

GROWTH AND SENESCENCE IN PUREBRED JERSEY COWS

BY F. A. DAVIDSON*

INTRODUCTION

The American Jersey Cattle Club early established a special register known as the Register of Merit, in which all purebred Jersey cows are eligible to entry upon the fulfilment of a minimum milk-production requirement. This requirement is based upon the pounds of butterfat produced during a single lactation and varies linearly with the age of the cows. Any purebred Jersey cow may have her production record entered in the Register of Merit as many times as she meets the requirement for the age at which her production is made. The cows that have only one record entered in the Register are spoken of as original-entry cows, those having more than one record entered are spoken of as reentry cows. Many thousands of entries have been made in the Register with the result that a large body of data on the milk production of Jersey cows has accumulated. These production records, however, are not quite representative of the production records of the purebred cows composing the breed as a whole owing to the selective and environmental influences imposed upon the cows entered in the Register.

Gowen (1920) pointed out that the production requirement made by the Register of Merit eliminates many of the low-producing cows of the breed, which results in the truncation of the yearly butterfat-yield frequency distributions of the Register cows at successive ages. Gowen, altho recognizing this selective effect of the production requirement, made no effort to correct for it and instead used the production records of a single herd of purebred Jersey cows as representative of the productions of the cows in the breed as a whole.

It is a common practice among breeders to provide the best possible environment, from the standpoint of both growth and production, for the cows they intend to submit for entry in the Register of Merit. Hence the reentry cows have a better chance to develop than the original-entry cows, since they are subjected for more than one lactation to

*This investigation was started by Mr. Davidson when a member of the Dairy Department of the University of Illinois and was completed by him under the direction of Professor Sewall Wright at the University of Chicago. The author wishes to express his appreciation to Professor Wright for his guidance and many valuable suggestions during the progress of the study.

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an environment highly stimulating to growth. Kildee and McCandlish (1916), Eckles (1918, 1920), and McCandlish (1920) all have demonstrated very clearly the influence of environment upon the rate of growth and milk secretion in dairy cows. Cows provided with the best possible environment show a distinctly superior rate of growth and milk production. There is also an inclination among breeders to select only the highest-producing cows in their herds to submit for reentry in the Register of Merit. Such a practice would have a tendency to bring about a genetic superiority of the reentry cows over the original-entry cows from the standpoint of milk production. This difference between the reentry and original-entry cows, altho never actually demonstrated by the breeders, has long been recognized by them, and as will be shown later, is not without justification.

Since the reentry cows are kept under an environment which gives them a better chance to develop than the original-entry cows and since the former are also subject to selection by the breeders, it does not seem logical to lump the records of both the reentry and original-entry cows together when using them to make a study of the course of growth and senescence in purebred Jersey cows. Many investigators—Pearl, Gowen, and Miner (1919), Hooper (1921), Brody, Ragsdale, and Turner (1923a), Turner, Ragsdale, and Brody (1924), and Graves and Fohrman (1925)—have used these Register-of-Merit records for this purpose, but with the exception of Graves and Fohrman, all have lumped the original-entry and reentry records together in their studies. The latter have been the only investigators to separate the original-entry from the reentry records and study them separately. The results from this investigation, which will be discussed in more detail in another section, show that there is a marked difference between the original-entry and reentry cows, as measured by the rise and fall in their yearly butterfat productions with advancing age.

THE PROBLEM

In this study the Register-of-Merit records of the original-entry and reentry Jersey cows have been analyzed separately by means of biometrical methods from the following points of view:

1. The course of growth in Register-of-Merit Jersey cows as described by the increase in their body weights with advancing age.
2. The course of growth and senescence in Register-of-Merit Jersey cows as described by the rise and fall in their yearly butterfat yields with advancing age. This involved the preliminary problem of correcting for the truncation of the yearly butterfat frequency dis-

tributions at successive ages, due to the selective effect of the Register-of-Merit production requirement.

3. A comparison of the course of growth and senescence in the original-entry and reentry cows.

SOURCE OF DATA

The Register-of-Merit records involved in this study consist of all of the 365-day original-entry and reentry records as published in the yearly volumes of the Register of Merit up to and inclusive of the volume for 1920. These volumes contain 9,694 original-entry records and 2,628 reentry records, or a total of 12,322 records in all. Each record as published in the Register of Merit includes the following items of information concerning the cow.

1. Age at the beginning of the lactation period
2. Weight at the beginning of the lactation period
3. Length of the record in days
4. Total milk yield
5. Total butterfat yield
6. Percentage of butterfat in the milk
7. Designation of entry, whether first, second, etc.

BIOMETRICAL ANALYSIS OF DATA

COURSE OF GROWTH IN REGISTER-OF-MERIT JERSEY COWS AS DESCRIBED BY INCREASE IN BODY WEIGHT WITH ADVANCING AGE

Dairy cows are peculiar in that they show very little, if any, tendency to fattening, this being especially true of Register-of-Merit Jersey cows. Hence the increase in the body weights of these cows with advancing age may be used as a fair measure of their rate of growth.

The frequency distributions of the body weights for the Register-of-Merit Jersey cows at successive ages following 1.5 years of age for the original-entry cows and 2.5 years of age for the reentry cows, are reported in Tables 1 and 2 and Figs. 1 and 2 respectively. The histograms in Figs. 1 and 2 all tend towards the symmetrical or normal type. These histograms, however, are not representative of the true type of frequency distribution of body weight, owing to the fact that the body weights of the cows are in part estimated. The original-entry records include 1,096 records for which actual body weights of the cows are listed. These records were analyzed separately and the frequency distributions of body weight for these cows are reported in Table 3 and the last four distributions in Fig. 1. The fitted histograms in Fig. 1 may be assumed to represent the true type of distribution of body weight for Jersey cows. It will be noticed that these frequency dis-

TABLE 1.—FREQUENCY DISTRIBUTIONS OF BODY WEIGHTS FOR ORIGINAL-ENTRY JERSEY COWS WITH ADVANCING AGE

Weight in pounds	Age in years																				
	1.5-2.0	2.0-	2.5-	3.0-	3.5-	4.0-	4.5-	5.0-	5.5-	6.0-	6.5-	7.0-	7.5-	8.0-	8.5-	9.0-	10.0-	11.0-	12.0-	13.0-18.5	
400-450																					
450-	3	7	4	1	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1
500-	6	8	17	11	3	4	1	10	6	4	2	2	2	6	4	3	5	5	5	5	1
550-	11	102	179	205	5	18	12	29	14	16	11	15	9	25	19	18	12	8	2	3	1
600-	88	373	139	113	41	40	27	59	70	49	38	35	22	25	22	23	8	4	3	4	3
650-	158	497	189	133	136	105	82	67	74	47	40	30	27	26	22	23	31	19	14	9	6
700-	198	676	275	167	147	133	101	104	94	72	65	58	58	35	33	42	31	19	14	6	4
750-	153	341	186	185	147	133	101	104	94	72	65	58	58	35	33	42	31	19	14	6	4
800-	174	276	169	185	147	133	101	104	94	72	65	58	58	35	33	42	31	19	14	6	4
850-	42	79	60	77	45	83	60	51	48	56	40	46	28	21	20	43	11	6	11	7	7
900-	13	45	33	50	47	63	64	57	60	58	47	42	42	32	37	33	29	10	7	7	7
950-	5	7	3	14	11	17	16	18	23	16	11	18	8	9	5	10	6	1	1	1	2
1000-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1050-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1100-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1150-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1200-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1250-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1300-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1350-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1400-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1450-	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
1500-1550	1	2	5	3	5	6	9	15	15	10	8	5	9	7	3	13	6	1	1	1	2
Total.....	831	2565	1086	905	636	585	456	419	413	335	264	253	206	164	146	191	116	50	39	34	34

Weight in pounds

TABLE 2.—FREQUENCY DISTRIBUTIONS OF BODY WEIGHTS FOR REENTRY JERSEY COWS WITH ADVANCING AGE

Weight in pounds	Age in years																		
	2.5-3.0	3.0-	3.5-	4.0-	4.5-	5.0-	5.5-	6.0-	6.5-	7.0-	7.5-	8.0-	8.5-	9.0-	10.0-	11.0-	12.0-	13.0-18.5	
600-650																			
650-	1	3	6	3	1	3	7	5	7	4	2	3	1	4	1				
700-	3	15	11	13	8	32	12	15	15	12	10	6	5	2	5				
750-	3	40	42	35	35	37	38	29	19	16	20	27	17	17	13	2			
800-	11	49	46	63	69	59	59	36	32	28	29	27	17	18	15	4			
850-	5	59	62	39	39	43	33	33	29	26	21	20	24	22	15	8			
900-	4	38	34	48	37	46	39	53	29	38	27	21	20	30	6	7			
950-	1	5	3	18	15	19	18	14	12	12	10	8	5	7	2	4			
1000-	1	4	2	10	10	14	11	11	12	13	14	7	6	9	2	2			
1100-			1	3	3	1	3	4	5	2	1	4	1	1		1			
1150-			1	1		2		4	4	2	4	1	3	1		4			
1200-																			
1250-								1	1	1			1						
1300-																			
1350-																			
1400-																			
1450-					1														
1500-1550																			
Total.....	35	245	238	285	247	260	242	194	169	158	133	101	78	118	49	38	19		19

TABLE 3.—FREQUENCY DISTRIBUTIONS OF ACTUAL BODY WEIGHTS FOR ORIGINAL-ENTRY JERSEY COWS WITH ADVANCING AGE

Weight in pounds	Age in years														
	1.5-2.0	2.0-	2.5-	3.0-	3.5-	4.0-	4.5-	5.0-	6.0-	7.0-	8.0-	9.0-	10.0-	11.0-	12.0-13.0
550-600	1	2	1	2											
600-	3	14	3	2											
650-	7	26	6	7	4	4	2	4	1	9	3				
700-	9	37	18	13	8	10	5	6	6	11	3				
750-	18	46	15	17	20	17	8	18	4	24	11	2	3	1	
800-	11	52	24	17	21	10	7	16	9	24	12	2	8	2	3
850-	6	45	12	12	13	10	6	10	8	11	6	4	3	2	1
900-	2	29	13	18	7	11	6	18	15	11	6	4	2	3	
950-	2	18	11	8	7	6	4	6	5	7	4	5	3	3	
1000-		9	3	3	4	2	4	6	5	7	2	2	1	3	
1050-		3	2	3	4	2	4	6	5	7	2	2	1	3	
1100-		1	2		2	1	4	3	1	1	2	1	1	3	
1150-					2	1	2	3	1		2	1	1	3	
1200-															
1250-															
1300-1350															
Total.....	60	282	110	96	74	76	58	102	60	76	45	26	21	6	4

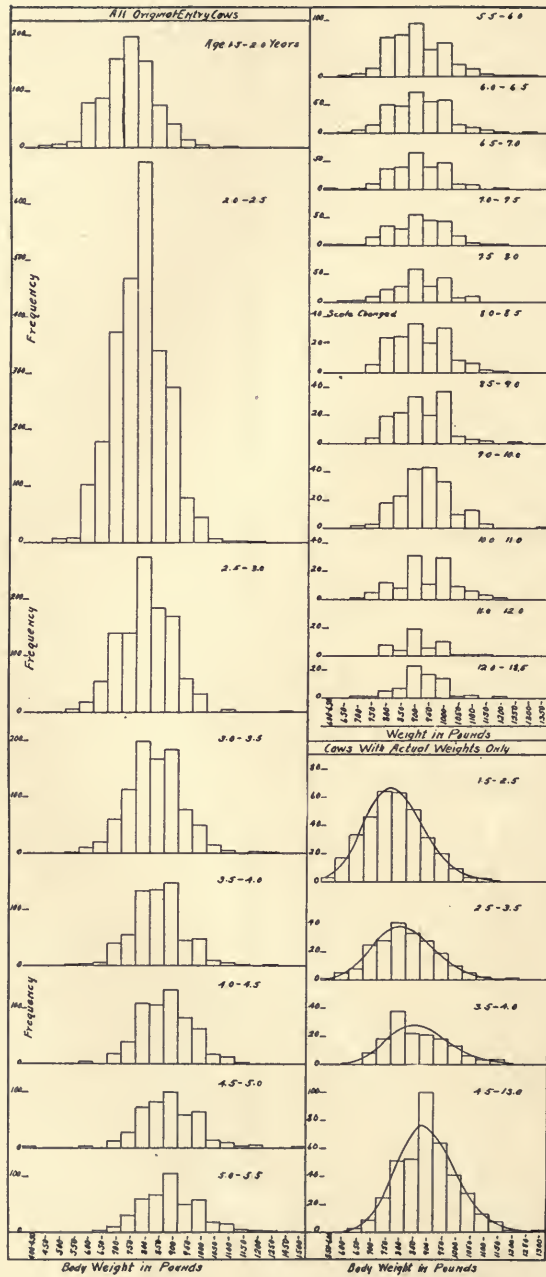


FIG. 1.—FREQUENCY DISTRIBUTIONS OF BODY WEIGHTS FOR ORIGINAL-ENTRY JERSEY COWS

tributions are not symmetrical but are skewed in the positive direction. A survey of the literature reveals the fact that the frequency distributions of the body weights and the weights of the parts of the bodies of animals, and even the weights of plant seeds, are of this type, that is, skewed in the positive direction.

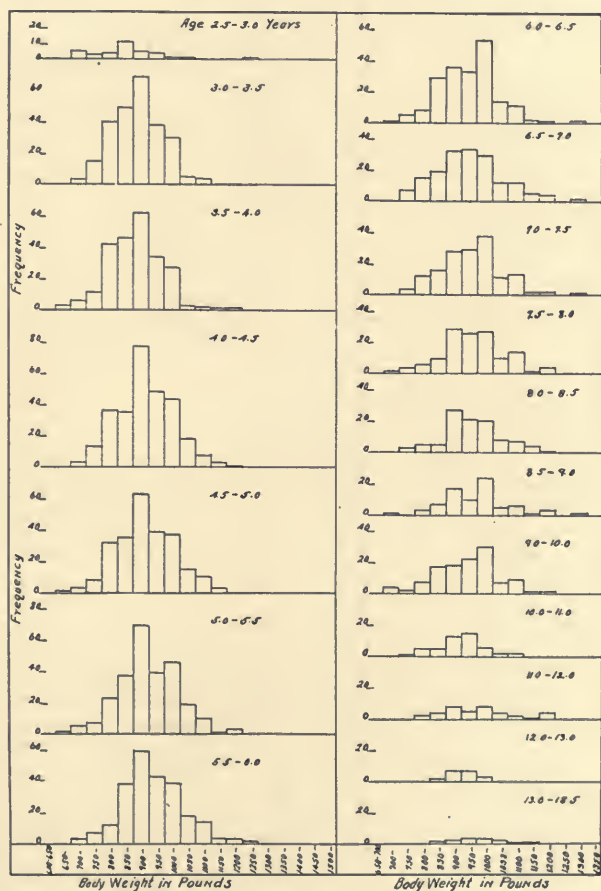


FIG. 2.—FREQUENCY DISTRIBUTIONS OF BODY WEIGHTS FOR REENTRY JERSEY COWS

When the frequency distributions of the actual body-weight data are transformed to the logarithmic basis, that is, instead of the body weight the logarithm of the body weight is used as the variable, these distributions tend to become symmetrical and of the normal type. This peculiarity of these frequency distributions suggests that the

TABLE 4.—CONSTANTS OF NORMAL AND LOG-TRANSFORMED FREQUENCY CURVES FITTED TO ACTUAL BODY-WEIGHT DATA FOR ORIGINAL-ENTRY JERSEY COWS

Age in years	Fre- quency	a	s	Mean, in pounds	Standard deviation	n^*		χ^2		P^{**}	
						Log curve	Normal curve	Log curve	Normal curve	Log curve	Normal curve
1.5-2.5	342	2.9060 ± .0020	.05561 ± .0014	810 ± 3.79	104 ± 2.68	7	7	2.5197	5.0052	.92	.59
2.5-3.5	206	2.9250 ± .0026	.05513 ± .0018	851 ± 5.07	108 ± 3.59	6	6	1.9809	3.4040	.92	.76
3.5-4.5	150	2.9513 ± .0028	.05165 ± .0020	893 ± 5.67	103 ± 4.01	5	5	11.5285	15.4993	.08	.01
4.5-12.5	398	2.9664 ± .0017	.04906 ± .0012	934 ± 3.55	105 ± 2.51	8	8	13.6497	18.4857	.09	.02

* n = the number of $(c-c)^2$ terms corrected for the loss of 3 degrees of freedom, the latter being the total frequency, the mean, and the standard deviation. n' in Pearson's tables = $n + 1$.

**A value of P equal to .10 or greater may be considered as representing a good fit. The larger the value of P , the closer is the agreement between the observed and fitted frequencies.

TABLE 5.—STATISTICS OF BODY WEIGHTS FOR ORIGINAL-ENTRY JERSEY COWS WITH ADVANCING AGE (Estimated and actual weights)

Age in years	Mean in pounds		Standard deviation		Coefficient of variation	
	Estimated + actual**	Actual	Estimated + actual	Actual	Estimated + actual	Actual
1.5-2.0	766 ± 2.13	782 ± 8.10	91 ± 1.50	93 ± 5.73	11.9 ± .197	11.9 ± .733
2.0-	808 ± 1.22	816 ± 4.22	92 ± .86	105 ± 2.98	11.3 ± .106	12.9 ± .366
2.5-	836 ± 1.90	834 ± 7.07	93 ± 1.34	110 ± 5.00	11.1 ± .161	13.2 ± .600
3.0-	867 ± 2.11	869 ± 7.16	94 ± 1.49	104 ± 5.06	10.9 ± .173	11.9 ± .579
3.5-	881 ± 2.46	893 ± 8.31	92 ± 1.74	106 ± 5.88	10.4 ± .197	11.9 ± .660
4.0-	906 ± 2.55	893 ± 7.74	91 ± 1.80	100 ± 5.47	10.1 ± .199	11.2 ± .613
4.5-	922 ± 3.30	937 ± 10.63	104 ± 2.33	120 ± 7.51	11.3 ± .252	12.8 ± .802
5.0-	925 ± 3.26	*926 ± 7.81	99 ± 2.31	*117 ± 5.52	10.7 ± .249	*12.7 ± .600
5.5-	931 ± 3.22	*947 ± 9.40	97 ± 2.28	*108 ± 6.65	10.4 ± .244	*11.4 ± .702
6.0-	937 ± 3.51	933 ± 3.65	95 ± 2.48	98 ± 2.58	10.2 ± .266	10.1 ± .553
6.5-	933 ± 3.65	*917 ± 7.20	88 ± 2.58	*93 ± 5.09	9.4 ± .276	*10.1 ± .553
7.0-	934 ± 4.02	940 ± 4.08	95 ± 2.84	97 ± 2.89	10.2 ± .306	*10.0 ± .711
7.5-	940 ± 4.91	*932 ± 9.35	87 ± 3.47	*93 ± 6.61	9.2 ± .306	9.7 ± .369
8.0-	944 ± 4.91	944 ± 5.29	93 ± 3.74	95 ± 3.74	9.9 ± .369	10.0 ± .711
8.5-	944 ± 4.56	965 ± 7.94	95 ± 3.22	93 ± 3.22	10.0 ± .395	9.7 ± .369
9.0-	961 ± 6.08	944 ± 11.04	97 ± 4.30	74 ± 7.81	9.7 ± .447	6.3 ± .589
10.0-	964 ± 7.43	958	78 ± 5.25	78 ± 5.25	10.1 ± .447	7.9 ± .822
11.0-	943	938	78 ± 5.25	78 ± 5.25	8.3 ± .560
12.0-13.0	960 ± 6.62	938	61 ± 4.68	61 ± 4.68	6.4 ± .489

*The age mid-points of these statistics are 5.5, 6.5, 7.5, and 8.5 respectively.

**The estimated plus the actual weights include the weights of all original-entry cows.

factors affecting the body weights of these cows tend to have constant percentage effects, rather than constant absolute effects, thruout the entire scale. McAlister (1879) has shown that this type of frequency distribution can be fitted by the log-transformed equation of the normal frequency curve, the equation of the curve being $y =$

$$\frac{1}{sW\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\log W - a}{s}\right]^2}$$

in which $W =$ body weight, $y =$ the ordinate, and a and s are the mean and standard deviation on the log scale. In order to make certain that the frequency distributions of the actual body-weight data were of the above-mentioned skewed type, they were fitted by both the normal frequency curve and the log-transformed frequency curve, the constants of which are reported in Table 4. In this table are also included the χ^2 and probability values (P) measur-

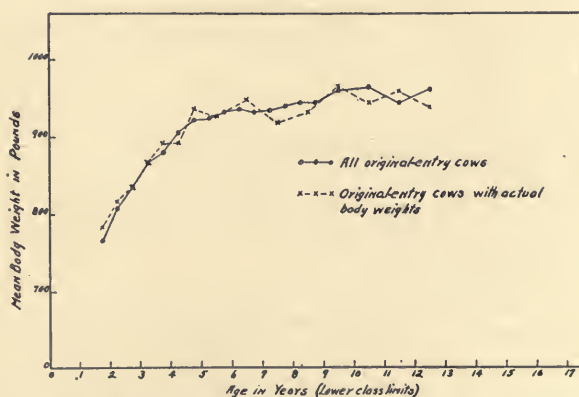


FIG. 3.—MEAN BODY WEIGHTS OF ORIGINAL-ENTRY COWS

ing the goodness of fit of these curves. In every case it will be found that the log-transformed frequency curve gives the better fit to the data. The fitted curves in Fig. 1 are the log-transformed curves.

Altho the frequency distributions of the estimated plus the actual body-weight data do not represent the true type of frequency distribution of body weight, the means of these distributions show practically the same trend with advancing age as do the means of the actual body-weight distributions (see Table 5 and Fig. 3). Gowen (1925) made a comparison between the estimated and actual weights of purebred Holstein cows and found that the means and standard deviations were practically the same in the two sets of data. Hence owing to the

TABLE 6.—STATISTICS OF BODY WEIGHTS FOR ORIGINAL-ENTRY AND REENTRY JERSEY COWS WITH ADVANCING AGE

Age in years	Mean in pounds		Standard deviation		Coefficient of variation	
	Original-entry	Reentry	Original-entry	Reentry	Original-entry	Reentry
1.5-2.0	766 ± 2.13	91 ± 1.50	11.9 ± .197
2.0-	808 ± 1.22	92 ± 1.86	11.3 ± .106
2.5-	836 ± 1.90	870 ± 12.44	83 ± 1.84	109 ± 8.80	11.3 ± .161	12.6 ± 1.016
3.0-	867 ± 2.11	911 ± 3.61	94 ± 1.49	84 ± 2.55	10.9 ± .173	9.2 ± .280
3.5-	881 ± 2.40	907 ± 3.73	92 ± 1.74	85 ± 2.94	10.4 ± .197	9.4 ± .291
4.0-	906 ± 2.55	940 ± 3.59	91 ± 1.80	90 ± 2.54	10.1 ± .199	9.6 ± .271
4.5-	922 ± 3.30	942 ± 4.25	104 ± 2.33	99 ± 3.01	11.5 ± .232	10.5 ± .319
5.0-	925 ± 3.26	949 ± 3.90	99 ± 2.31	93 ± 2.76	10.7 ± .249	9.8 ± .290
5.5-	931 ± 3.22	964 ± 4.21	97 ± 2.28	97 ± 2.98	10.4 ± .244	10.1 ± .310
6.0-	937 ± 3.51	973 ± 4.38	95 ± 2.48	90 ± 3.10	10.2 ± .266	9.3 ± .318
6.5-	933 ± 3.65	973 ± 5.47	88 ± 2.58	106 ± 3.87	9.4 ± .276	10.8 ± .396
7.0-	934 ± 4.02	982 ± 5.42	95 ± 2.84	101 ± 3.83	10.2 ± .306	10.3 ± .391
7.5-	940 ± 4.08	983 ± 5.86	87 ± 2.89	100 ± 4.14	9.9 ± .306	10.2 ± .422
8.0-	944 ± 4.91	982 ± 6.09	93 ± 3.47	91 ± 4.31	9.9 ± .369	9.2 ± .437
8.5-	944 ± 5.29	996 ± 7.71	95 ± 3.74	101 ± 5.45	10.0 ± .395	10.1 ± .545
9.0-	961 ± 4.56	996 ± 6.07	93 ± 3.22	98 ± 4.29	9.7 ± .335	10.1 ± .443
10.0-	964 ± 6.08	949 ± 7.15	97 ± 4.30	74 ± 5.06	10.1 ± .447	7.8 ± .531
11.0-	943 ± 7.43	1007 ± 11.98	78 ± 5.25	110 ± 8.47	8.3 ± .560	10.9 ± .843
12.0-13.0	960 ± 6.62	954 ± 6.42	94 ± 4.68	42 ± 4.54	6.4 ± .489	4.4 ± .482
14.1*	950 ± 11.38	98 ± 8.05	10.3 ± .843
14.5*	1012 ± 13.14	85 ± 9.30	8.4 ± .920

*Average age of cows ranging from 13.0 to 18.5 years of age.

advantages provided by the larger numbers, the estimated and the actual weights were combined in studying the course of growth as described by the increase in body weight with advancing age in both the original-entry and reentry cows.

The statistics of the frequency distributions of body weight for both the original-entry and reentry cows are reported in Table 6. Altho the mean body weights of the reentry cows are much greater than the mean body weights of the original-entry cows, the coefficients of variation are very much the same in both sets of data. Hence the most significant difference between these two groups of cows lies in the greater size of the reentry cows.

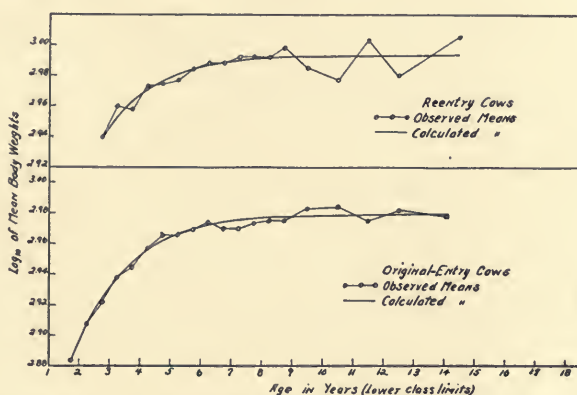


FIG. 4.—LOG₁₀ OF OBSERVED AND CALCULATED MEAN BODY WEIGHTS OF REENTRY AND ORIGINAL-ENTRY COWS

The mean body weights of the original-entry and reentry cows reported in Fig. 4 increase with advancing age up to the age of approximately 8 years, where they reach their maximum and after which they remain more or less constant. The course of development of these Register-of-Merit cows as described by the increase in their body weight with advancing age may be expressed in the form of a mathematical equation representing their rate of growth. Accordingly the next step in the analysis consists in the selection of a growth curve which best represents the data and which also has a logical interpretation from the biological point of view.

Growth Curves Applied to Individual Growth in Animals

Review of Literature.—It was early recognized that growth in animals is cyclic in nature and does not consist of a single uninterrupted progression from birth to maturity. Each growth cycle consists first of a period of slow growth,

followed by a period of relatively rapid growth, which in turn is succeeded by a period of slow growth. The curve representing a growth cycle is of the rising and falling type and tends to be symmetrical around its center, the maximum of the cycle. This cyclic nature of growth has been described by Minot (1891) and Read (1912) in guinea pigs, Donaldson (1915) in the rat, Ostwald (1908) and Robertson (1916) in mice, Robertson (1923) in man, and Brody and Ragsdale (1922) in cattle, sheep, and other domestic animals. According to the theories of growth developed by Loeb (1906), Ostwald (1908), and Robertson (1923), growth under normal conditions is limited by a series of consecutive autocatalytic monomolecular chemical reactions and the middle, or maximum, of each growth cycle represents the middle of the respective limiting chemical reaction. The equation of the autocatalytic monomolecular chemical reaction as derived by Robertson (1923) is $\frac{dx}{dt} = k_1 x (A - x)$.

A in this equation represents the ultimate amount of growth attainable in the cycle in question and theoretically determines the quantity of the growth-promoting substance present at the beginning of the cycle and exhausted during its course. x is proportional to the amount of this substance converted at any time t , and k_1 is a constant determining the velocity of growth. On plotting $\frac{dx}{dt}$ for various values of x , a symmetrical curve of the rising and falling type is obtained with a maximum where $x = \frac{1}{2}A$. When integrated this equation assumes

the form of $\log \frac{x}{A-x} = KA(t - t_1)$ which when plotted gives an S curve with a point of inflection at its center. The formula embodied in this equation has been applied with more or less success by Robertson (1923) and Brody (1922) to the growth cycles in man, mice, rats, guinea pigs, rabbits, cattle, sheep, swine, and chickens.

Brody and Ragsdale (1921) have shown that growth in the dairy cow consists of two extrauterine cycles and one intrauterine cycle. Data on purebred Holstein cows and purebred Jersey cows were presented to show the extrauterine cycles. The first extrauterine cycle commences slightly before birth, reaches its maximum at about 5 months of age and continues to the age of 15 months. The second cycle begins immediately after the first, reaches its maximum at about 20 months of age and continues to the age of maturity. Hence growth in dairy cows, after the age of 2 years, is non-cyclic in nature and follows an uninterrupted progression to maturity. Brody next turned his attention to the growth of Register-of-Merit Jersey cows (Brody, Ragsdale, and Turner, 1923a), combining the weights of the original-entry and reentry cows. These data, as has been shown, represent the course of growth after the age of 2 years and are non-cyclic in nature. A quotation from Brody will best express his views in regard to these data. "The data show that after the age of 2 years the rate of growth declines in a non-cyclic manner. The course of decline in growth follows the course of decline of a monomolecular chemical reaction; that is, the percentage decline in growth with age is constant." The equation of the monomolecular chemical reaction used by Brody *et al* is $W = A(1 - e^{-kt})$ where A represents the weight of the animal at maturity, W the weight of the animal at any time t , and k the velocity constant of growth. This equation when applied to the data gives a fairly good fit. Later Brody and Ragsdale (1924) attempted to reach a growth

equation representing the whole course of development from birth to maturity. The Department of Animal Husbandry of the University of Missouri has for years been collecting a large number of linear measurements and body weights of purebred Jersey cows at intervals from birth to maturity. These data were used by Brody to represent the whole course of development in Jersey cows from birth to maturity. The cyclic fluctuations are obvious in these data, but in the present study they were not considered. The growth equation derived by Brody to fit the data is $W = A - Be^{-kt}$ where W is the weight or linear dimension at any time t , A is the limit reached at maturity, k is the velocity constant of growth, and B is a constant locating the curve in point of time. This equation has the same form as the equation of a monomolecular chemical reaction with the exception that in a monomolecular reaction A and B have the same value and the curve begins at zero. Brody's interpretation of the curve is as follows: "Barring fluctuations due to the cyclic phenomena, the extrauterine course of growth in linear dimensions and in weight of the dairy cow follows an exponential law having the same form as the law representing the course of monomolecular change in chemistry. This suggests the interpretation that the general course of growth is limited by a monomolecular chemical process, and that the cyclic phenomena are due to subsidiary processes in the fundamentally exponential course of growth. . . . This is in accordance with expectations if it is assumed that each animal begins life with a definite endowment of limiting substance necessary for the process of growth, and that this endowment is used up at a constant rate (or percentage) of itself." In a later paper Brody (1926) gives a somewhat different interpretation. "One may, of course, with equally good logic, interpret this equation as indicating the production during the course of growth of a growth-retarding substance according to the monomolecular law."

Altho there seems to be a striking similarity between the course of growth in animals and the course followed by a monomolecular chemical reaction, it seems doubtful whether such a complicated process as growth would follow so simple a chemical reaction. The same criticism holds true for the autocatalytic monomolecular theory of growth. The S curve of the autocatalytic monomolecular reaction is a rather flexible curve and can be made to approximate closely a great many growth reactions in both animals and plants. However, the point of inflection of this curve is at its center, whereas most of the growth reactions show a point of inflection earlier in the reaction. Van de Sande-Bakhuyzen and Alsborg (1927) have given a very thoro criticism of the autocatalytic monomolecular chemical theory as applied to growth in animals and plants and have presented evidence to show that the reactions involved in growth cannot be represented by such a simple chemical theory.

A growth curve similar to that derived by Brody (1924) but with a more general biological meaning may be derived in the following manner. Minot (1908) showed for a number of animals that the percentage increments in body weight, $\frac{W_2 - W_1}{W_1}$, constantly decrease from birth to maturity. These percentage increments may be looked upon as measuring the average growth power of the body cells, if growth power may be defined as the percentage rate of increase in growth.

Wright (1926) suggested briefly that the hypothesis that growth power falls off at a constant percentage rate leading to the curve $\log \log \frac{c}{w} = a - kt$ might often give a good fit to growth data. This curve may also be expressed in the form $\log W = A - be^{-kt}$, curiously similar to Brody's formula. The derivation of this equation is as follows:

$$\frac{dW}{Wdt} = P \quad (1)$$

where W = body weight at any time t , and P = growth power of the body cells. Since the growth power is assumed to fall off at a constant percentage rate,

$$\frac{dP}{Pdt} = -k \quad (2)$$

$$\log P = C - kt$$

$$P = e^{C-kt} = \frac{dW}{Wdt} \quad (3)$$

$$\begin{aligned} \log W &= -\frac{1}{k} e^{C-kt} + A \\ &= A - \frac{e^C}{k} e^{-kt} \\ &= A - be^{-kt} \end{aligned} \quad (4)$$

In equation (4) A is the logarithm of the weight of the animal at maturity; $100k$ is the constant percentage rate of decrease in growth power on the above interpretation, and b locates the curve in time; W is the weight at any time t . This equation differs from Brody's growth equation in that W is replaced by $\log W$ and A is the logarithm of weight instead of the actual weight at maturity. Also, it does not involve any simple chemical interpretation of growth. The curve for weight (W) is S-shaped with the point of inflection at $\frac{1}{e} = 37$ percent of the final weight.

We may now turn to the growth data of the original-entry and re-entry Register-of-Merit Jersey cows presented in Table 6. Equation (4) was applied to these data, and the values calculated from the fitted

equations are reported in Table 7. The fitted equations for original-entry and reentry cows are respectively:

$$\log_{10} W = 2.9793 - .127344e^{-.2763t}$$

$$\log_{10} W = 2.9930 - .134378e^{-.2993t}$$

In these equations t = age and is measured in units of six months beginning with one year and three months as the origin. W = weight at any age t .

TABLE 7.—MEAN BODY WEIGHTS FOR ORIGINAL-ENTRY AND REENTRY JERSEY COWS WITH ADVANCING AGE*

Age in years	Original-entry			Reentry		
	Mean observed	Log ₁₀ Mean observed	Log ₁₀ Mean calculated	Mean observed	Log ₁₀ Mean observed	Log ₁₀ Mean calculated
1.5-2.0	766	2.88423	2.88270
2.0-	808	2.90741	2.90602
2.5-	836	2.92211	2.92371	870	2.93952	2.93826
3.0-	867	2.93802	2.93713	911	2.95952	2.95242
3.5-	881	2.94498	2.94731	907	2.95761	2.96292
4.0-	906	2.95713	2.95503	940	2.97313	2.97070
4.5-	922	2.96473	2.96089	942	2.97405	2.97647
5.0-	925	2.96614	2.96534	949	2.97727	2.98074
5.5-	931	2.96895	2.96871	964	2.98408	2.98391
6.0-	937	2.97174	2.97126	973	2.98811	2.98626
6.5-	933	2.96988	2.97320	973	2.98811	2.98801
7.0-	934	2.97035	2.97468	982	2.99211	2.98930
7.5-	940	2.97313	2.97579	983	2.99255	2.99026
8.0-	944	2.97497	2.97664	982	2.99211	2.99097
8.5-	944	2.97497	2.97728	996	2.99826	2.99149
9.0-	961	2.98272	2.97795	966	2.98498	2.99202
10.0-	964	2.98408	2.97852	949	2.97727	2.99246
11.0-	943	2.97451	2.97885	1007	3.00303	2.99271
12.0-13.0	960	2.98227	2.97904	954	2.97955	2.99283
14.1*	950	2.97772	2.97920
14.5*	1012	3.00518	2.99299

The equations of the curves fitted to observed values are: original-entry, $\log_{10} W = 2.9793 - .127344e^{-.2763t}$; reentry, $\log_{10} W = 2.9930 - .134378e^{-.2993t}$.

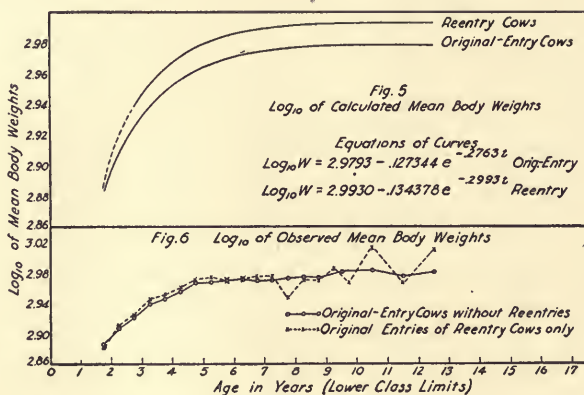
*Average age of cows ranging from 13.0 to 18.5 years of age.

The smooth curves in Fig. 4 describing the course of growth in body weight of the original-entry and reentry Jersey cows are the fitted-growth curves represented by the above equations. It will be noted that the trends of these curves closely agree with the trends of the observed mean body weights with advancing age. Therefore it may be assumed that growth power in body weight of the original-entry and reentry Jersey cows after 2 years of age is falling off at a fairly constant percentage rate. It should be noted that this curve cannot be carried back to birth on this basis.

Comparison of Course of Growth in Body Weight of Original-Entry and Reentry Cows

The smooth curves in Fig. 4 describing the course of growth in the original-entry and reentry cows are plotted together for compar-

ison in Fig. 5. These curves, when compared, show that on the average the reentry cows are distinctly larger and increase in weight more rapidly than do the original-entry cows. The same relation is indicated in the growth constants of the equations of these curves. The A constant, which is the logarithm of the body weight at maturity, is greater for the reentry cows than for the original-entry cows, the values of these constants being 2.9930 and 2.9793 respectively. The greater value of k for the reentry cows than for the original-entry cows indicates a more rapid rate of growth in the former, the values of these constants being $-.2993$ and $-.2763$ respectively.



FIGS. 5 AND 6.—COMPARISON OF BODY-WEIGHT CURVES OF ORIGINAL-ENTRY AND REENTRY COWS

The graphs in Fig. 6 represent the mean body weights of the original-entry cows that do not have reentry records and the mean body weights of the original entries of the reentry cows only, that is, the mean body weights of the reentry cows when they made their first or original-entry record. The mean body weights of the original entries of the reentry cows are not significantly different from the mean body weights of the other original-entry cows. In view of this close agreement between the mean body weights, it is not likely that the reentry cows are selected for reentry on account of their superior body size. Hence it may be assumed that the greater size and more rapid rate of growth found in the reentry cows is due largely to the more favorable environment under which they are kept. This conclusion is in agreement with the experimental work of Eckles and Swett (1918) wherein they describe a difference in the course of growth between heavy-fed Jersey cows and light-fed Jersey cows, similar to that evidenced between reentry and original-entry Jersey cows.

Since increase in weight with age may be due to an accumulation of inert substances within the body cells rather than to an increase in the mass of physiologically-active protoplasm within them, it is desirable to supplement the body-weight data with growth measurements that bear directly upon the increase in the mass of physiologically active tissue with advancing age. The primary function of a dairy cow is the secretion of milk, that is, all of her energy in excess of the requirements for maintenance, gestation excluded, is expended in the production of milk. Hence any change in the activity of the mammary gland with advancing age will reflect in a general way a similar change in the physiological activity of other organs of the body.

The Register-of-Merit cows, as previously stated, must meet a minimum production requirement which varies with their age. This production requirement naturally eliminates many of the lower-producing cows of the breed and hence, if these Register-of-Merit production records are to be used to describe the course of growth and senescence in Jersey cows, some correction must be made for the records of the cows eliminated by the requirement. So far no one has attempted to estimate the number of cows eliminated by the requirement. Most authors have considered it of minor importance, assuming that the requirement eliminated only a small percentage of all the cows making the Register of Merit.

It will be remembered that this production requirement is levied upon the butterfat yields of the cows; therefore it was necessary to study the butterfat yields rather than the total milk yields of the cows. This, however, causes no serious disturbance in the interpretation, because the butterfat content of Jersey milk contains in the neighborhood of 60 percent of the total energy of the yield.

TRUNCATION OF YEARLY BUTTERFAT FREQUENCY DISTRIBUTIONS DUE TO SELECTIVE EFFECT OF REGISTER-OF-MERIT REQUIREMENT

The frequency distributions of the yearly butterfat yields at successive ages for the original-entry and reentry Register-of-Merit Jersey cows are reported in Tables 8 and 9 and Figs. 7 and 8 respectively. The histograms in Fig. 7 showing the frequency distributions of the yearly fat yields for the original-entry cows are severely truncated. The percentage of truncation, as will be shown later, ranges from 10 to 39. The corresponding histograms in Fig. 8 for the reentry cows, however, are only slightly truncated, as will likewise be shown later, ranging from 2 to 4 percent. This nearly complete lack of truncation of the reentry fat-yield distributions, as compared to original-entry

TABLE 8.—FREQUENCY DISTRIBUTIONS OF 365-DAY FAT YIELDS FOR ORIGINAL-ENTRY JERSEY COWS WITH ADVANCING AGE

Fat yield in pounds	Age in years																			
	1.5-2.0	2.0-	2.5-	3.0-	3.5-	4.0-	4.5-	5.0-	5.5-	6.0-	6.5-	7.0-	7.5-	8.0-	8.5-	9.0-	10.0-	11.0-	12.0-18.5	
250-300	230	451	89	16	104	44	7
300-	258	690	296	196	104	44	7
350-	186	672	298	244	183	143	103	86	78	64	51	40	38	23	28	32	20	10	...	23
400-	94	385	180	218	143	146	116	121	103	81	72	77	52	47	39	59	34	17	...	18
450-	34	193	113	120	92	125	104	90	93	67	48	49	43	36	36	42	27	10	...	18
500-	17	106	51	58	50	53	59	62	62	54	36	33	32	27	22	24	12	6	...	10
550-	9	38	37	25	29	48	37	26	35	31	28	23	16	14	11	12	12	3	...	1
600-	2	17	14	16	19	16	19	18	24	17	13	15	11	7	7	11	6	3	...	1
650-	5	7	11	5	5	4	11	11	7	8	4	5	2	5	4	1	...	2
700-	2	2	3	4	2	4	3	6	5	4	7	2	2	3
750-	1	1	2	1	2	2	3	2	2	1	2	1	2
800-
850-
900-
950-1000
Total.....	831	2565	1086	905	636	585	456	419	413	335	264	253	206	164	146	191	116	50	...	73

TABLE 9.—FREQUENCY DISTRIBUTIONS OF 365-DAY FAT YIELDS FOR REENTRY JERSEY COWS WITH ADVANCING AGE

Fat yield in pounds	Age in years																				
	2.5-3.0	3.0-	3.5-	4.0-	4.5-	5.0-	5.5-	6.0-	6.5-	7.0-	7.5-	8.0-	8.5-	9.0-	10.0-	11.0-	12.0-18.5				
250-300	1	1	15	6	1	
300-	7	26	31	32	20	20	21	11	10	9	11	11	1	3	3	6	2	
350-	6	38	45	43	39	36	37	27	27	17	14	18	6	14	9	3	8	
400-	9	45	33	57	42	65	42	31	28	24	16	10	9	21	8	6	8	
450-	1	45	33	47	45	39	31	31	30	26	24	17	24	18	8	3	6	
500-	7	35	36	47	45	39	31	31	30	26	24	17	24	18	8	3	6	
550-	3	26	35	39	35	32	31	27	27	23	17	16	16	16	8	4	7	
600-	1	15	15	24	28	29	28	27	27	23	18	17	9	11	7	4	3	
650-	...	10	15	11	19	19	22	16	14	16	14	11	12	12	7	3	1	
700-	...	6	7	10	18	11	15	8	4	6	9	17	10	10	3	2	1	
750-	...	3	1	7	6	6	5	4	10	5	4	3	4	3	..	1	1	
800-	...	3	2	4	4	2	6	3	8	4	4	3	..	1	1	3	1	
850-	...	1	1	4	1	1	4	1	3	2	2	3	..	1	1	1	1	
900-
950-
1000-1050
Total.....	35	245	238	285	247	260	242	194	169	158	133	101	78	118	49	38	38	

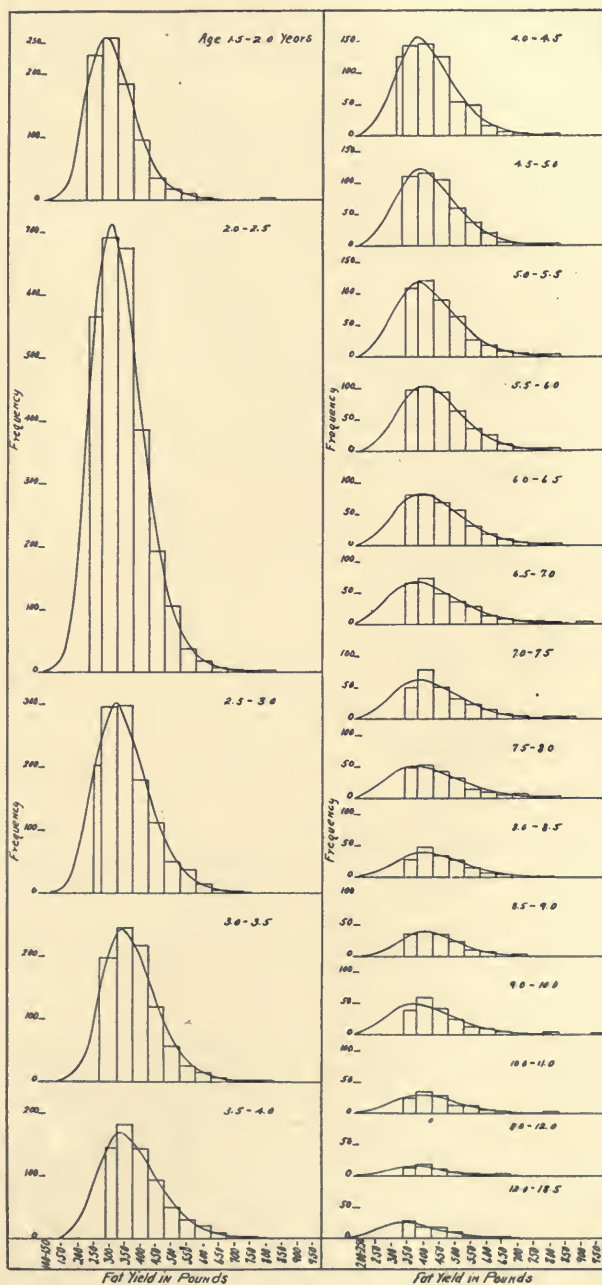


FIG. 7.—FREQUENCY DISTRIBUTIONS OF FAT YIELDS FOR ORIGINAL-ENTRY JERSEY COWS

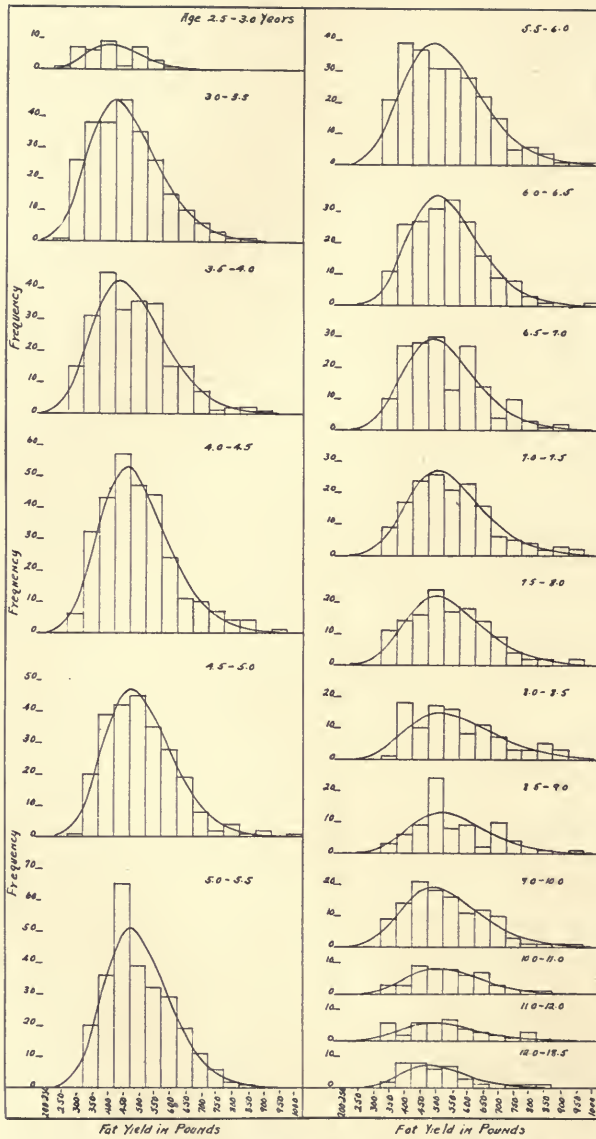


FIG. 8.—FREQUENCY DISTRIBUTIONS OF FAT YIELDS FOR REENTRY JERSEY COWS

fat-yield distributions, may be attributed to the high level of production of the reentry cows, that is, their production is apparently beyond that specified by the Register-of-Merit production requirement. Owing

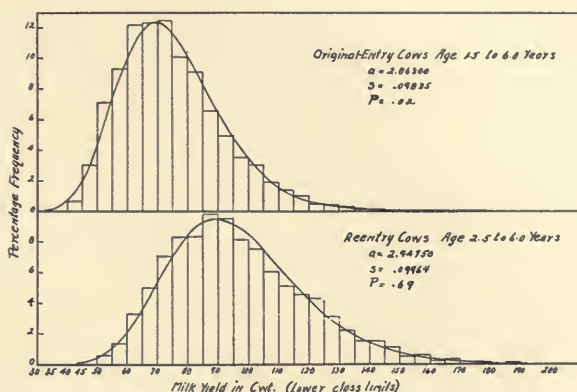


FIG. 9.—YEARLY MILK YIELDS OF REGISTER-OF-MERIT JERSEY COWS UNDER SIX YEARS

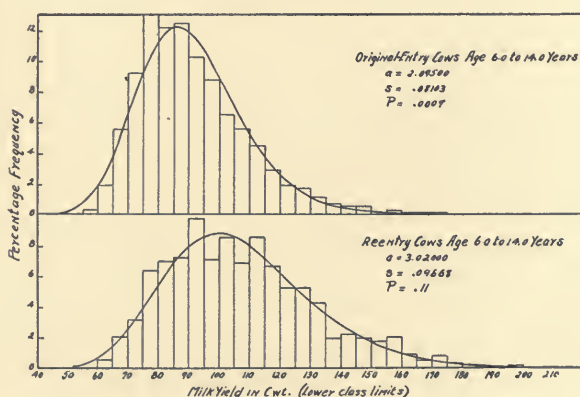


FIG. 10.—YEARLY MILK YIELDS OF REGISTER-OF-MERIT JERSEY COWS OVER SIX YEARS

to this practical completeness of the yearly fat-yield frequency distributions of the reentry cows, they were considered as the type representative of yearly fat-yield frequency distributions for all purebred Jersey cows.

A further analysis of these reentry fat-yield frequency distributions revealed the fact that they, like the actual body-weight frequency distributions, are not symmetrical but are skewed in the positive direction. Here again these fat-yield distributions when transformed to the logarithmic scale become symmetrical and can be described by the log-transformed equation of the normal frequency curve previously

cited. This peculiarity of these fat-yield frequency distributions suggests that the factors affecting yearly butterfat yield likewise tend to have a constant percentage effect rather than an absolute effect. This same peculiarity is also found in the frequency distributions of the yearly milk yields for the reentry cows reported in Figs. 9 and 10.

Before going further into a discussion of these reentry fat-yield frequency distributions, it might be well to give a brief discussion of the significance of the constants of this log-transformed frequency curve. Heretofore it has been assumed that the arithmetic mean and standard deviation were appropriate statistics to measure the variability in the milk yields of dairy cows. The use of the arithmetic mean and standard deviation presupposes that the frequency distribution of the data is symmetrical. This, however, as has been demonstrated, is not true in the case of milk and fat yield as well as body weight. Galton (1879) pointed out that the ordinary law of frequency of error based on the arithmetic mean demands that deviations in excess of the mean must be balanced by deviations of equal magnitude in deficiency. In other words, the frequency distributions must be symmetrical. He also pointed out that in some cases where the deviations in excess of the mean are greater than those in deficiency, the geometric mean and not the arithmetic mean best represents the mean of the data. McAlister (1879), who at Galton's suggestion gave a mathematical proof of the applicability of the geometric mean to such data, essentially assumed the existence of this peculiarity of the factors affecting the data. When the body-weight, milk-yield, and fat-yield frequency distributions are transformed to the log scale, they become symmetrical, and hence the arithmetic mean and the standard deviation are appropriate statistics to apply on this scale. The arithmetic

mean of the log scale is $\frac{\sum \log x}{n}$, which equals $\log \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \dots x_n}$,

which in turn is the logarithm of the geometric mean on the original scale. The standard deviation on the log scale, when interpreted on the original scale, approximates the logarithm of $(1 + \text{the coefficient of variation})$ and seems to be the best measure of the percentage variability in data of this type.

*Application of Log-Transformed Normal Frequency Curve to
Yearly Fat-Yield Frequency Distributions of
Reentry Cows*

The smooth curves describing the yearly fat-yield frequency distributions for the reentry cows in Fig. 8 are fitted log-transformed

frequency curves, the general equation of these curves being

$$y = \frac{1}{sx \sqrt{2\pi}} e^{-\frac{1}{2} \left[\frac{\log x - a}{s} \right]^2}$$

where x = the fat yield and a and s are the mean and standard deviation on the log scale. The means (a) and the standard deviations (s) of these fitted curves, together with the probability values (P) measuring their goodness of fit, are reported in Table 10. It will be noted that the probability values of the goodness of fit of these curves, with few exceptions, indicate that there is a remarkably close agreement between the observed and calculated frequencies in the distributions. In view of this exceptionally good fit of the log-transformed normal frequency curve to the reentry fat-yield frequency distributions, it was assumed that the frequency curve which best describes the yearly fat-yield frequency distributions characteristic of purebred Jersey cows is the log-transformed normal curve.

TABLE 10.—STATISTICS OF LOG-TRANSFORMED NORMAL CURVE FITTED TO FAT YIELDS OF REENTRY JERSEY COWS

Age in years	a	s	Frequency	n^*	χ^2	P^{**}
2.5-3.0	2.6460 ± .0098	.08594 ± .0069	35	3	10.1639	.02
3.0-	2.6750 ± .0044	.10150 ± .0031	245	7	2.0768	.95
3.5-	2.6915 ± .0044	.10090 ± .0031	238	7	7.4892	.38
4.0-	2.7075 ± .0037	.09346 ± .0026	285	7	6.3937	.50
4.5-	2.7220 ± .0038	.08783 ± .0027	247	6	2.3799	.88
5.0-	2.7175 ± .0036	.08645 ± .0026	260	6	10.4279	.11
5.5-	2.7340 ± .0044	.10120 ± .0031	242	8	8.4483	.39
6.0-	2.7400 ± .0043	.08862 ± .0030	194	7	4.1042	.77
6.5-	2.7315 ± .0049	.09531 ± .0035	169	7	20.1871	.01
7.0-	2.7440 ± .0050	.09300 ± .0035	158	7	5.2092	.64
7.5-	2.7410 ± .0056	.09623 ± .0040	133	7	2.6176	.91
8.0-	2.7590 ± .0072	.10690 ± .0051	101	7	6.0326	.54
8.5-	2.7525 ± .0071	.09290 ± .0051	78	6	20.7688	.002
9.0-	2.7375 ± .0062	.09966 ± .0044	118	6	4.4881	.61
10.0-	2.7440 ± .0090	.09300 ± .0063	49	4	2.8702	.58
11.0-	2.7410 ± .0118	.10740 ± .0083	38	3	1.9832	.58
13.4***	2.7250 ± .0094	.08621 ± .0067	38	2	2.2453	.33

*Number of $\frac{(o-c)^2}{c}$ terms corrected for the loss of 3 degrees of freedom, viz., total frequency, a and s .

**A value of P equal to .10 or greater may be considered as representing a close agreement between the observed and fitted frequencies. The larger the value of P , of course the closer the agreement.

***Average age of cows ranging from 12.0 to 13.5 years of age.

Correction for Truncation of Yearly Fat-Yield Frequency Distributions of Original-Entry Cows

The original-entry yearly fat-yield frequency distributions in Fig. 7, being severely truncated, do not represent the yearly fat-yield frequency distributions of all original-entry Jersey cows. However, now that a frequency curve has been found that is representative of the true type of frequency distribution of the yearly fat yields of

TABLE 11.—STATISTICS OF LOG-T-TRANSFORMED NORMAL CURVE FITTED TO THE FAT YIELDS OF ORIGINAL-ENTRY JERSEY COWS

Age in years	a	s	N'	N'' (estimated)	Fre- quency	Percentage truncation	π^*	χ^2	P^{**}
1.5-2.0	2.51050	± .0022	947.8	941.4	830	12.8	4	2.0602	.57
2.0-	2.54222	± .0013	2857.8	2864.5	2565	10.5	7	10.6699	.16
2.5-	2.56032	± .0020	1254.0	1251.2	1086	13.2	6	10.1026	.12
3.0-	2.58806	± .0020	1014.5	1015.8	904	11.0	6	6.4185	.38
3.5-	2.58795	± .0026	800.8	797.4	636	20.2	6	5.9873	.43
4.0-	2.62293	± .0024	686.2	680.6	585	14.0	6	10.4357	.11
4.5-	2.62984	± .0028	568.7	565.4	456	19.4	5	4.0267	.55
5.0-	2.62578	± .0029	556.3	553.4	419	24.3	6	3.3524	.76
5.5-	2.64362	± .0029	506.8	505.7	413	18.3	6	1.5519	.93
6.0-	2.63695	± .0034	425.9	431.0	335	22.3	5	.7448	.98
6.5-	2.63441	± .0041	371.2	367.5	264	28.2	5	2.8953	.82
7.0-	2.62707	± .0048	291.8	332.0	253	23.8	6	6.2633	.31
7.5-	2.62707	± .0049	208.8	204.3	206	27.8	6	7.1810	.31
8.0-	2.64535	± .0045	174.3	173.8	164	19.7	5	4.4343	.49
8.5-	2.64294	± .0045	174.3	173.8	146	16.0	4	7.9718	.91
9.0-	2.63731	± .0047	153.1	262.6	190	27.6	5	7.0775	.22
10.0-	2.63731	± .0058	73.7	146.9	115	21.7	4	4.3072	.37
11.0-12.0	2.61312	± .0085	129.2	71.1	50	29.6	3	1.4991	.69
13.0***	2.58522	± .0067	129.2	120.7	73	39.5	1	2.5986	.11

*Number of $(c-c)^2$ terms corrected for the loss of 3 degrees of freedom, viz., total frequency, a and s .

**See Table 10.

***Average age of cows ranging from 12.0 to 18.5 years of age.

Jersey cows, it is possible to fit this curve to the truncated data as far as they go and then by extrapolation derive a frequency curve which may be assumed to be representative of the yearly fat-yield frequency distributions of the population of which the original-entry cows are a truncated sample. By extrapolation of this curve it is also possible to estimate the area of the frequency distribution cut off by the truncation of the data. Pearson (1902) found the frequency distribution of the speed, that is, the time consumed in running one mile, of registered American trotting horses was truncated at the speed required by the American Association for registration of the horses. In order to estimate the frequency distribution of the speed for all American trotting horses, Pearson changed the normal frequency curve into the form of a parabola $y' = a + bx + cx^2$, where $y' = \log y$, by taking the logarithm of the frequencies instead of the actual frequencies themselves. In this form the normal frequency curve was fitted by the method of moments to the truncated data and by extrapolation a frequency curve was derived which Pearson assumed to be representative of the frequency curve of speed for all American trotters. In a similar manner the log-transformed equation of the normal frequency curve for Jersey cows was changed into the form of a parabola $y' = a + bx' + c(x')^2$ where $y' = \log y$ and $x' = \log x$, and by the method of least squares was fitted to the truncated original fat-yield frequency distributions. The method of fitting these curves is described in detail in the Appendix. The fitted curves in every case were extrapolated and by this procedure curves were derived intended to be representative of the untruncated populations. The means (a) and the standard deviations (s) of these fitted curves, the probability values (P) measuring their goodness of fit, and the percentage of truncation of the frequency distributions estimated from the extrapolation of these fitted curves are reported in Table 11. The probability values (P) of the goodness of fit of these curves indicate that there is a close agreement between the observed and fitted frequencies in the distributions. The fit of the smooth curves to the actual data may be seen in Fig. 7.

Truncation of Yearly Milk-Yield Frequency Distributions of Original-Entry Cows

The frequency distributions of the yearly milk yields for the original-entry and reentry Jersey cows are reported in Tables 12 and 13 and Figs. 9 and 10. The frequency distributions of the yearly milk yields for the reentry cows, as mentioned before, are of the same type

TABLE 12.—FREQUENCY DISTRIBUTIONS OF 365-DAY MILK YIELDS FOR ORIGINAL-ENTRY JERSEY COWS

Milk yield in cwt.	Age 1.5 to 6.0 years			Age 6.0 to 14.0 years		
	Frequency observed	Percentage frequency observed	Percentage frequency calculated	Frequency observed	Percentage frequency observed	Percentage frequency calculated
35-4040
40-	56	.71	1.25
45-	235	2.98	3.12
50-	560	7.09	5.8548
55-	733	9.28	8.79	5	.28	1.22
60-	962	12.20	11.13	34	1.91	2.84
65-	972	12.31	12.24	97	5.46	5.23
70-	989	12.53	12.12	163	9.17	7.98
75-	794	10.06	10.94	232	13.06	10.38
80-	718	9.09	9.18	216	12.16	11.85
85-	523	6.62	7.31	221	12.44	12.10
90-	388	4.92	5.49	182	10.24	11.31
95-	278	3.52	4.00	156	8.77	9.76
100-	233	2.95	2.79	116	6.53	7.80
105-	148	1.88	1.91	100	5.62	6.00
110-	109	1.38	1.26	79	4.45	4.37
115-	78	.99	.82	51	2.87	3.05
120-	42	.53	.53	34	1.91	2.06
125-	30	.38	.33	30	1.68	1.40
130-	19	.24	.21	20	1.13	.85
135-	9	.11	.13	12	.68	.53
140-	9	.11	.08	10	.56	.32
145-	4	.05	.05	9	.50	.19
150-	1	.01	.02	3	.17	.12
155-	2	.03	.02	4	.23	.07
160-	1	.01	.01	1	.06	.04
165-01	1	.06	.02
170-	1	.01	.01	1	.06	.03
175-180	1	.01	.00
Total.....	7895	100.00	100.00	1777	100.00	100.00
<i>a</i>		2.86300 ± .0008			2.09500 ± .0013	
<i>s</i>		.09835 ± .0005			.08103 ± .0009	
<i>P</i>		.02			.0007	

as the butterfat distributions. The smooth curves describing the milk-yield distributions for the reentry cows in Figs. 9 and 10 are the fitted log-transformed normal curves. The probability values (*P*) measuring the goodness of fit of these curves indicate that there is a close agreement between the observed and fitted frequencies. The smooth curves describing the milk-yield distributions for the original-entry cows in Figs. 9 and 10 are likewise the log-transformed normal curves. The probability values (*P*) measuring the goodness of fit of these curves, however, indicate that there is a very poor agreement between the observed and fitted frequencies. It will be noted that the original-entry milk-yield distributions do not have as great a range as the reentry milk-yield distributions, the standard deviations (*s*) of the reentry fitted curves being considerably greater than the standard deviations of the original-entry fitted curves. The lack of range in the original-entry milk-yield distributions can, for the most part, be attributed to the uniform truncation of these distributions to the left of their modes. The original-entry fat-yield distributions, as previously

TABLE 13.—FREQUENCY DISTRIBUTIONS OF 365-DAY MILK YIELDS FOR REENTRY JERSEY COWS

Milk yield in cwt.	Age 2.5 to 6.0 years			Age 6.0 to 14.0 years		
	Frequency observed	Percentage frequency observed	Percentage frequency calculated	Frequency observed	Percentage frequency observed	Percentage frequency calculated
50-55	9	.58	.94
55-	21	1.35	1.5062
60-	51	3.29	2.80	6	.56	.99
65-	77	4.96	4.46	21	1.98	1.91
70-	109	7.02	6.21	33	3.11	3.17
75-	128	8.25	7.76	67	6.31	4.64
80-	129	8.31	8.89	73	6.87	6.11
85-	152	9.79	9.40	76	7.16	7.38
90-	148	9.54	9.32	103	9.70	8.29
95-	126	8.12	8.82	74	6.97	8.69
100-	117	7.54	7.95	90	8.47	8.68
105-	94	6.06	6.84	72	6.78	8.27
110-	79	5.09	5.76	91	8.57	7.56
115-	71	4.57	4.69	70	6.59	6.66
120-	67	4.32	3.71	55	5.18	5.73
125-	48	3.09	2.86	55	5.18	4.75
130-	34	2.19	2.20	45	4.24	3.86
135-	23	1.48	1.65	20	1.88	3.10
140-	23	1.48	1.21	23	2.17	2.40
145-	17	1.10	.88	20	1.88	1.87
150-	8	.52	.63	18	1.70	1.42
155-	9	.58	.46	21	1.98	1.05
160-	4	.26	.32	9	.85	.80
165-	5	.32	.23	5	.47	.58
170-16	8	.75	.42
175-	2	.13	.11	3	.28	.30
180-08	2	.19	.23
185-	1	.06	.16	1	.09	.14
190-11
195-200	1	.09	.27
Total.....	1552	100.00	100.00	1062	100.00	100.00
\bar{a}		2.94750 ± .0017			3.02000 ± .0020	
s		.09964 ± .0012			.09668 ± .0014	
P		.69			.11	

shown, are abruptly truncated by the Register-of-Merit production requirement and the milk-yield distributions would be similarly truncated if there existed a perfect correlation between milk yield and fat yield. Gowen (1919) found a correlation of +.89 between the milk yields and fat yields of purebred Holstein cows and surely a similar correlation exists for Jersey cows. The uniform truncation of the original-entry milk-yield distributions decreases their range and likewise the goodness of fit of the log-transformed normal curves applied to them.

COMPARISON OF MEANS (\bar{V}) AND STANDARD DEVIATIONS (σ) ON ORIGINAL SCALE WITH MEANS (a) AND STANDARD DEVIATIONS (s) ON LOGARITHMIC SCALE

Since the arithmetic means of the yearly fat yields of the Register-of-Merit Jersey cows have been used extensively to demonstrate the rise and fall in the rate of milk secretion of the cows with advancing age, it seems best to show the relation which exists between these

means and the corresponding geometric mean fat yields (arithmetic means on the log scale). The logarithms of the arithmetic mean and the logarithms of the geometric mean (values of a) yearly fat yields at successive ages for both the original-entry and reentry cows are graphed in Fig. 11. It will be noted that for the reentry cows the arithmetic mean fat yields lie slightly above and tend to parallel the geometric mean fat yields. Since the reentry yearly fat-yield frequency distributions, from which these statistics were derived, are not truncated much, it may be assumed that on the average the arithmetic mean fat yields will be only slightly greater and tend to parallel the geometric means of the same data. The arithmetic mean fat yields

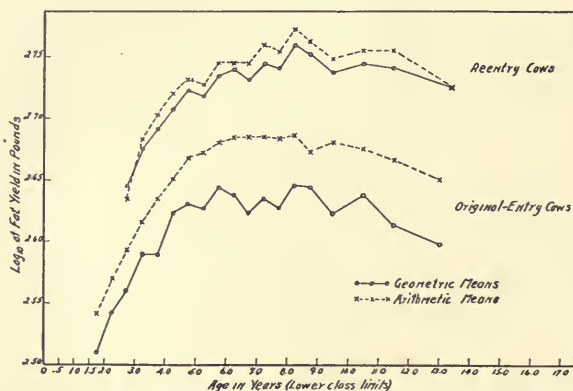


FIG. 11.—ARITHMETIC AND GEOMETRIC MEAN BUTTERFAT YIELDS OF REGISTER-OF-MERIT JERSEY COWS

of the original-entry cows, however, bear an altogether different relation to the corresponding geometric mean fat yields. The difference between these arithmetic and geometric mean fat yields is greater than that found for reentry cows and besides markedly increases with the age of the cows. It will be remembered that the percentage truncation of the original-entry fat-yield frequency distributions increases with the age of cows from 10 to 39 percent. Hence this increasing difference between the arithmetic mean and geometric mean fat yields of the original-entry cows reflects the percentage truncation of the related fat-yield frequency distributions. It is very evident that the rate of milk secretion of the original-entry cows with advancing age follows a distinctly different course when described by the arithmetic means of the truncated fat-yield data than when described by the geometric

TABLE 14.—STATISTICS OF FAT YIELDS FOR ORIGINAL-ENTRY AND REENTRY JERSEY COWS WITH ADVANCING AGE

Age in years	Mean in pounds		Standard deviation		Coefficient of variation	
	Original-entry	Reentry	Original-entry	Reentry	Original-entry	Reentry
1.5-2.0	348 ± 1.58	68 ± 1.12	19.5 ± .33
2.0-	371 ± 1.96	77 ± 1.68	20.6 ± .19
2.5-	391 ± 1.64	432 ± 10.17	80 ± 1.16	89 ± 7.19	20.5 ± .29	20.6 ± 1.73
3.0-	412 ± 1.81	482 ± 4.81	81 ± 1.28	112 ± 3.40	19.6 ± .32	23.1 ± .74
3.5-	430 ± 2.30	504 ± 5.10	86 ± 1.63	117 ± 3.61	20.1 ± .39	23.2 ± .75
4.0-	448 ± 2.24	524 ± 4.70	80 ± 1.58	118 ± 3.33	17.9 ± .37	22.5 ± .67
4.5-	465 ± 2.61	538 ± 4.88	83 ± 1.84	114 ± 3.45	17.7 ± .41	21.1 ± .67
5.0-	469 ± 2.73	534 ± 4.36	83 ± 1.93	104 ± 3.08	17.7 ± .43	19.5 ± .60
5.5-	478 ± 2.83	556 ± 5.63	85 ± 2.00	130 ± 3.98	17.9 ± .43	23.3 ± .75
6.0-	483 ± 3.38	555 ± 5.52	92 ± 2.39	114 ± 3.90	19.0 ± .51	20.5 ± .73
6.5-	483 ± 3.94	556 ± 6.29	95 ± 2.79	121 ± 4.45	19.7 ± .61	21.8 ± .83
7.0-	484 ± 3.99	576 ± 7.00	94 ± 2.82	130 ± 4.95	19.4 ± .60	22.6 ± .90
7.5-	482 ± 4.47	569 ± 7.38	95 ± 3.16	126 ± 5.22	19.8 ± .69	22.2 ± .96
8.0-	485 ± 4.74	592 ± 9.33	90 ± 3.35	139 ± 6.60	18.6 ± .71	23.5 ± 1.18
8.5-	479 ± 4.15	579 ± 9.73	74 ± 2.94	127 ± 6.88	15.8 ± .64	22.0 ± 1.24
9.0-	476 ± 4.65	560 ± 7.69	95 ± 3.29	124 ± 5.44	17.9 ± .71	22.1 ± 1.01
10.0-	476 ± 5.29	568 ± 10.57	85 ± 3.74	110 ± 7.47	17.8 ± .81	19.3 ± 1.36
11.0-12.0	463 ± 7.11	570 ± 15.33	75 ± 5.03	140 ± 10.84	16.1 ± 1.11	24.6 ± 2.01
13.0*	447 ± 5.32	67 ± 3.76	15.1 ± .86
13.3*	530 ± 12.31	113 ± 8.70	21.2 ± 1.71

*Average age of cows ranging from 12.0 to 18.5 years of age.

means derived from the fitted and extrapolated log-transformed frequency curves.

The standard deviations on the original scale of the yearly fat yields of both the original-entry and reentry cows (Table 14) tend to rise and fall with advancing age of the cows. The coefficients of variability of these fat yields (Table 14), on the other hand, practically remain constant. Gowen (1924) describes a similar relation between the standard deviations and the coefficients of variability of the yearly fat yields of purebred Holstein cows. The standard deviations of the yearly fat yields on the log scale derived from the fitted log-transformed curves, for both the original-entry and reentry cows (Tables 10 and 11) also practically remain constant. Since the standard deviation on the log scale, as previously stated, approximated the logarithm of $(1 + \text{the coefficient of variation})$, it is to be expected that the trend in these standard deviations (s) with advancing age should be similar to the trend in the corresponding coefficients of variation on the original scale.

Now that the average yearly fat yields for all original-entry and reentry Register-of-Merit cows have been determined, the former free from the effects of truncation, the next step consists in the study of the course of growth and senescence in Jersey cows as described by the rise and fall in their yearly fat yields with advancing age.

COURSE OF GROWTH AND SENESCENCE AS DESCRIBED BY RISE AND FALL IN YEARLY BUTTERFAT YIELDS WITH ADVANCING AGE

The geometric means, values of a , of the yearly fat yields for both the original-entry and reentry cows (Figs. 12 and 13) tend to increase but at an ever-decreasing rate with advancing age up to a maximum, which is attained at the age of maximum production of the cows. Upon reaching the age of maximum productivity, the mean fat yields change in trend and tend to decrease at an ever-increasing rate as age increases. It will be noted that up to the age of maximum production the mean fat yields follow a course similar to that followed by the mean body weights, and therefore supplement the body weights as a measure of growth in the cows. Brody, Ragsdale, and Turner (1923b) found a similar relation between the arithmetic mean fat yields and body weights of purebred Jersey cows. The mean fat yields not only supplement the body weights but also provide a better measurement of growth with respect to the physiologically-active body tissue.

The mean body weights, after attaining their maximum at approximately the same age as the mean fat yields, remain more or less

constant, and hence no longer reflect the effect of age on the body tissues of the cows. The mean fat yields, however, after the age of maximum production, take a downward course and steadily decline as age advances. This steady decline in the mean fat yields after

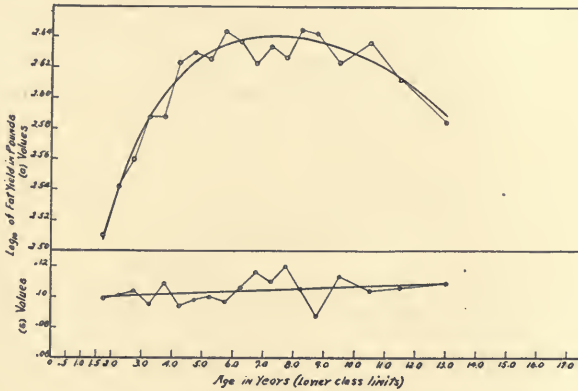


FIG. 12.—GEOMETRIC MEANS (a) AND STANDARD DEVIATIONS (s) OF YEARLY BUTTERFAT YIELDS FOR ORIGINAL-ENTRY COWS

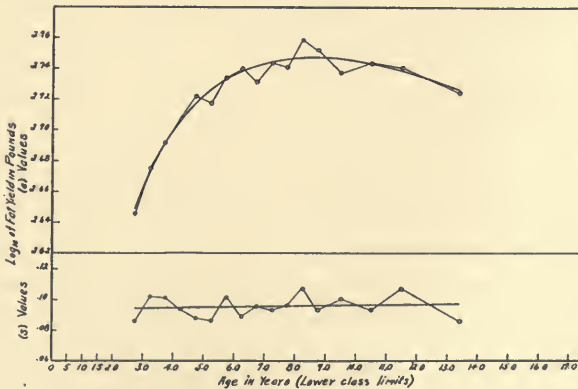


FIG. 13.—GEOMETRIC MEANS (a) AND STANDARD DEVIATIONS (s) OF YEARLY BUTTERFAT YIELDS FOR REENTRY COWS

the age of maximum production may therefore be used as a measure of the influence of age on the physiological activity of the cows during senility. Brody *et al* (1923c) referred to this decline in the yearly fat yields during senility as a measure of senescence in the cows; in other words, the process of senescence was interpreted as apply-

ing only to the decline in the physiological activity of the cows after the age of maturity. Gowen (1924) also gives a similar interpretation to this decline in the rate of milk secretion after the age of maximum production. Such an interpretation of the process of senescence would lead one to believe that it is initiated by the onset of senility and is represented by the decline in the physiological activity of the cells during old age. Altho the evidence of senescence becomes very marked during senility, the process as a whole needs a broader and more general interpretation than that given by Brody and Gowen.

Senescence the Result of Same Processes Which Determine Growth and Differentiation

Review of Literature.—Minot (1908) defined senescence as the process of growing old and associated it with the fundamental processes of growth and differentiation. He presented evidence based upon the growth increments of rabbits, guinea pigs, and chicks to show that the rate of growth is highest in the young organism and decreases as development proceeds and the rate of metabolism falls. He considers the rate of growth as a measure of the rate of senescence and therefore concludes that the rate of senescence is highest in youth and slowest in advanced life. Minot explains the phenomenon of senescence on the basis of the cytoplasmic changes taking place with age and presents evidence to show that growth and differentiation of the cytoplasm are the fundamental factors in senescence and death. He finds that in the young cell the amount of cytoplasm in relation to the amount of nuclear substance is least, but that during development the cytoplasm increases in proportion to the nuclear material, undergoes differentiation, and brings about senescence. Minot, however, did not explain how these changes in the cytoplasm bring about senescence. Child (1915) agrees with Minot in regarding the decrease in the growth power of the cells with advancing age as evidence of senescence, and also associates it with the cytoplasmic changes in the cells as age increases. Child, however, finds the cause of senescence in the ever-increasing mass of inactive protoplasm in the cells accompanying growth and differentiation. As the mass of inactive protoplasm increases, the mass of active protoplasm decreases, and hence the relative rate of metabolism decreases, which in turn brings about a decrease in the reproductive power of the cells. Child, therefore, regards senescence as "primarily a decrease in the rate of the dynamic processes conditioned by the accumulation, differentiation and other associated changes of the material colloid substratum." Consequently senescence is an inevitable feature of growth and differentiation and is not limited to the senile stages in the life cycle.

In the light of the more general interpretation of senescence, it appears logical to assume that the whole course of milk secretion in purebred Jersey cows from youth to old age is an expression of the senescent changes accompanying the growth and differentiation of their mammary glands. One of the outstanding features of senescence,

according to the above view, is the continuous decrease in the reproductive capacity (growth power) of the cells with advancing age. Such a process must necessarily be taking place in the mammary glands if their course of growth up to the age of maximum production may be described by the ever-decreasing rate of increase in the mean fat yields. Another outstanding feature of senescence, and according to Child the cause of this continuous decrease in the growth power of the cells, is the increase in the mass of inactive protoplasm in the cells accompanying growth and differentiation. As the mass of inactive protoplasm in the cells increases, the mass of active protoplasm decreases and likewise the relative rate of metabolism and functional activity of the cells. In other words, the relative functional activity of the cells in a gland constantly decreases as age increases. During the earlier stages of growth in a gland there is a rapid increase in the number of functional cells and the total functioning capacity of the gland increases regardless of the loss due to the decrease in the relative functional activity of the component cells. However, a point is ultimately reached where the increase in the number of functional cells does not compensate for the decrease in the relative functional activity of the component cells and the total functioning of the gland begins to decrease. Such a process would bring about a change from a rising to a falling trend in the total functioning of the gland with advancing age. Considering the function of the mammary gland from this standpoint, a similar process must necessarily be taking place within it, if the rise and fall in the mean fat yields may be used as a measure of its functional activity with advancing age. Hence the process of senescence is indicated just as clearly in the ever-decreasing rate of increase in the mean fat yields up to the age of maximum production as in the ever-increasing rate of decrease in the mean fat yields beyond the age of maximum production. In other words, the process of senescence is an inevitable consequence of development and its evidences are ever present regardless of whether the organism is in the growing or senile phases of the life cycle.

The course of growth and senescence in these purebred Register-of-Merit cows, as described by the rise and fall in their yearly fat yields with advancing age, may also be expressed in the form of a mathematical equation, providing an equation can be derived which not only represents the trend of the data but which also may be interpreted upon the basis of the biological phenomena involved in the more general theory of growth and senescence.

*Curves Describing Course of Growth and Senescence
as Measured by Lactation in Dairy Cows*

Review of Literature.—Pearl (1919) and Gowen (1924) have used the logarithmic equation, $y = a + bx + cx^2 + d \log x$ in which y = milk yield and x = age, to represent the rate of milk secretion with advancing age in all of the purebred breeds of dairy cattle. This equation was fitted with a great deal of accuracy to both the yearly milk yields and butterfat yields of Register-of-Merit Jersey cows. Altho this equation accurately represents the trend of the activity of the mammary gland as age increases, it is not based upon any general biological law, and hence may be considered as one of many empirical equations which change from an increasing to a decreasing trend with increasing values of the variables. Brody (1923c) combined the yearly fat yields, at successive ages, of all the purebred breeds of dairy cattle, including the milking Shorthorns, and found that the equation of two simultaneous consecutive monomolecular chemical reactions could be fitted to the data with some degree of accuracy. This equation takes the form of $M = A (ae^{-k_1 t} - be^{-k_2 t})$ in which k_2 and k_1 are the velocity constants of growth and senescence respectively, M = the fat yield and t = age. In order to give a biochemical interpretation to this equation, Brody and his coworkers assume that growth and senescence go on simultaneously from the beginning to the end of life, which assumption is not in harmony with their earlier statement. Their interpretation of this chemical equation is very clearly stated in the following quotation: "The whole course of milk secretion with age was therefore found to follow approximately the course of two simultaneous consecutive monomolecular reactions. This is taken to mean that growth and senescence go on simultaneously from the beginning to the end of life, and that each follows an exponential law with age; and therefore perhaps that the course of the two processes are limited by two consecutive chemical reactions." Altho there is a similarity between the course of milk secretion with advancing age and the course of two consecutive monomolecular chemical reactions, it seems rather absurd to assume that such a complicated physiological function as milk secretion is due to two simple chemical reactions. Furthermore, this equation separates growth (k_2) and senescence (k_1) into two distinct processes, a separation that cannot be justified according to the more generally accepted conception of senescence.

If it may be assumed that the increasing trend in the yearly fat yields up to the age of maximum production, for both the original-entry and reentry cows, reflects in the main the growth of their mammary glands, then the growth equation of the type used describing the increase in body weight, $\log_{10} M = A - be^{-kt}$, where M = fat yield, may be used to describe this trend in the yearly fat yields. Beyond the age of maximum productivity, however, the yearly fat yields of the cows change in trend and follow a decreasing course as age increases. This declining trend in the yearly fat yields after the age of maximum production may be assumed to represent the effect of senility in the mammary glands of the cows. Hence if the above growth equation is to be used to represent the whole course of milk secretion with advancing age, a corrective term must be added to it in order to ac-

count for this decline in the fat yields during senility. After the addition of a corrective term, this growth equation takes the form of $\log_{10} M = A - be^{-kit} - de^{kst}$ in which M = the fat yield and t = age measured in units of 6 months, beginning with 1 year 3 months as the origin. The significance of the other constants will be brought out in the following discussion. The first part of the above equation $\log_{10} M = A - be^{-kit}$ may be interpreted in a similar manner as previously described under the section on growth in body weight, and broadly speaking represents the increase in fat yields due to the growth of the mammary gland. The function of the mammary gland, however, depends not only upon the number of cells composing it, but also upon the relative physiological activity of the cells. The physiological activity of the cells depends upon the amount of active protoplasm within them, and since this constantly decreases with age and apparently at an ever-increasing rate, their relative physiological activity likewise decreases. Hence the corrective term de^{kst} that increases at an ever-increasing rate as age increases, must be subtracted from $\log_{10} M = A - be^{-kit}$ in order to account for this decrease in the relative physiological activity of the cells in the mammary gland accompanying growth and senility. This equation may be derived as follows: letting M be the milk production, N the number of secreting cells, and A a measure of the physiological activity of the cells in producing milk,

$$M = NA \text{ or } \log M = \log N + \log A \quad (5)$$

The rate of change in the milk production per unit of tissue would then be

$$\frac{dM}{Mdt} = \frac{dN}{Ndt} + \frac{dA}{Adt} \quad (6)$$

where t = time. The percentage change in the number of cells may be considered as a measure of the growth power of the cells (P), that is,

$$\frac{dN}{Ndt} = P \quad (7)$$

For convenience the percentage rate of decrease in the physiological activity per cell may be called S , that is,

$$-\frac{dA}{Adt} = S \quad (8)$$

Accordingly, then

$$\frac{dM}{Mdt} = P - S \quad (9)$$

If the growth power of the cells (P) falls off at a uniform percentage rate, then

$$\frac{dP}{Pdt} = -k_1$$

$$P = C_1 e^{-k_1 t} \quad (10)$$

If the percentage rate of loss in physiological activity in milk secretion per cell (S) is increasing at a uniform percentage rate, then

$$\frac{dS}{Sdt} = k_2$$

$$S = C_2 e^{k_2 t} \quad (11)$$

$$\frac{dM}{Mdt} = C_1 e^{-k_1 t} - C_2 e^{k_2 t}$$

which after integration may be put in the form of

$$\log M = A - be^{-k_1 t} - de^{k_2 t} \quad (12)$$

In this equation k_1 and k_2 are both constants determining the velocity of the senescence process. k_1 performs this function by determining the percentage rate of decrease in the growth power of the cells and k_2 by determining the percentage rate of increase in the percentage rate of loss in the relative physiological activity of the cells.^a This equation differs from Brody's equation in that it involves a growth equation of the type used to describe growth in body weight and furthermore is based upon a broad biological rather than a chemical theory of development.

The above equation (12) representing the course of growth and senescence during development was applied to the geometric mean fat yields at successive ages for both the original-entry and reentry Jersey cows. The formulas of these fitted equations are respectively:

$$\log_{10} M = 2.67567 - .203800e^{-.2349t} - .0059618e^{.1133t}$$

$$\log_{10} M = 2.7733 - .236561e^{-.2303t} - .0045624e^{.0953t}$$

in which M = fat yields and t = age in units of six months, beginning with 1 year 3 months as the origin. The values calculated from these fitted equations are reported in Tables 15 and 16 respectively. The

^aThe term $de^{k_2 t}$ is approximately $d + k_2 t$ when k_2 is small. Thus dk_2 is the approximate velocity constant of the percentage rate of decline in the physiological activity of the cells.

TABLE 15.—MEANS (*a*) AND STANDARD DEVIATIONS (*s*) OF YEARLY FAT YIELDS AT SUCCESSIVE AGES FOR ORIGINAL-ENTRY JERSEY COWS

Age in years	Log ₁₀ geometric mean (<i>a</i>)		Standard deviation on log ₁₀ scale (<i>s</i>)	
	Observed*	Calculated**	Observed*	Calculated***
1.5-2.0	2.51050	2.50784	.09895	.09982
2.0-	2.54222	2.54077	.10134	.100231
2.5-	2.56032	2.56653	.10405	.100643
3.0-	2.58806	2.58663	.09524	.101056
3.5-	2.58795	2.60218	.10866	.101469
4.0-	2.62293	2.61410	.09369	.101882
4.5-	2.62984	2.62311	.09790	.102295
5.0-	2.62578	2.62978	.09964	.102708
5.5-	2.64362	2.63452	.09668	.103120
6.0-	2.63695	2.63769	.10570	.103533
6.5-	2.62338	2.63954	.11602	.103946
7.0-	2.63441	2.64027	.10954	.104359
7.5-	2.62707	2.64003	.11994	.104772
8.0-	2.64535	2.63892	.10467	.105184
8.5-	2.64294	2.63701	.08708	.105597
9.0-	2.62339	2.63267	.11307	.106217
10.0-	2.63731	2.62440	.10366	.107042
11.0-12.0	2.61312	2.61301	.10627	.107868
13.0****	2.58522	2.58920	.10575	.108694

*The observed values of *a* and *s* are the constants of the log-transformed frequency curves fitted to the yearly fat-yield frequency distributions in Fig. 7.

**The calculated values of *a* were derived from the equation $\log_{10} M = 2.67567 - .2038e^{-.2340t} - .0059618e^{.1133t}$ where *t* = age in units of 6 months with the origin at 1 year 3 months and $\log_{10} M$ = values of *a*.

***The calculated values of *s* were derived from the equation $s = .099405 + .00041282t$, where *t* = age in units of 6 months beginning with 1 year 3 months as the origin.

****Average age of cows ranging from 12.0 to 18.5 years of age.

TABLE 16.—MEANS (*a*) AND STANDARD DEVIATIONS (*s*) OF YEARLY FAT YIELDS AT SUCCESSIVE AGES FOR REENTRY JERSEY COWS

Age in years	Log ₁₀ geometric mean (<i>a</i>)		Standard deviation on log ₁₀ scale (<i>s</i>)	
	Observed*	Calculated**	Observed*	Calculated***
2.5-3.0	2.6460	2.64865	.08594	.09386
3.0-	2.6750	2.67224	.10150	.09400
3.5-	2.6915	2.69114	.10090	.09414
4.0-	2.7075	2.70580	.09346	.09429
4.5-	2.7220	2.71721	.08783	.09443
5.0-	2.7175	2.72603	.08645	.09457
5.5-	2.7340	2.73276	.10120	.09472
6.0-	2.7400	2.73780	.08860	.09486
6.5-	2.7315	2.74149	.09531	.09500
7.0-	2.7440	2.74405	.09300	.09515
7.5-	2.7410	2.74569	.09623	.09529
8.0-	2.7590	2.74656	.10690	.09544
8.5-	2.7525	2.74676	.09290	.09558
9.0-	2.7375	2.74596	.09966	.09579
10.0-	2.7440	2.74330	.09300	.09608
11.0-12.0	2.7410	2.73895	.10740	.09637
13.4****	2.7250	2.72571	.08621	.09694

*The observed values of *a* and *s* are the constants of the log-transformed frequency curves fitted to the yearly fat-yield frequency distribution in Fig. 8.

**The calculated values of *a* were derived from the equation $\log_{10} M = 2.7733 - .236561e^{-.2303t} - .0045624e^{.0933t}$, where *t* = age in units of 6 months, with the origin at 1 year 3 months and $\log_{10} M$ = values of *a*.

***The calculated values of *s* were derived from the equation $s = .093714 + .000143418t$, where *t* = age in units of 6 months, beginning with 2 years 9 months as the origin.

****Average age of cows ranging from 12.0 to 18.5 years of age.

smooth curves in Figs. 12 and 13 describing the rise and fall in the yearly fat yields of the original-entry and reentry cows are the fitted growth and senescence curves represented by the above equations.

It will be noted that the trends of the fitted curves are in fair agreement with the trends of the observed mean fat yields with advancing age. Therefore it may be assumed that the rise and fall of the yearly fat yields of the original-entry and reentry Jersey cows is in agreement with the biological theory of growth and senescence involved in the equation $\log M = A - be^{-k_1t} - de^{k_2t}$ and hence is representative of the same processes in the development of the cow.

Comparison of Course of Growth and Senescence in Original-Entry and Reentry Cows

The smooth curves in Figs. 12 and 13 describing the course of milk secretion (yearly fat yields) with advancing age in the original-entry and reentry Jersey cows are plotted together for comparison in Fig. 14. A comparison of these curves shows that there is a marked

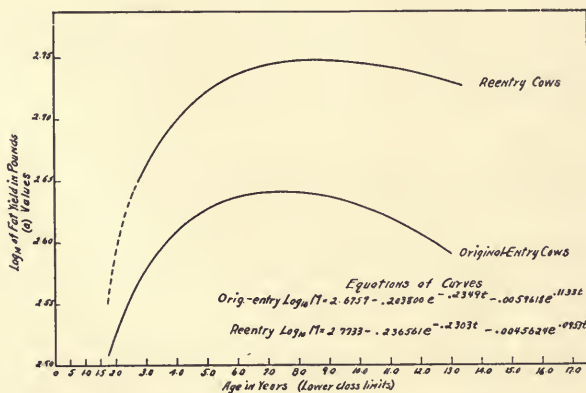


FIG. 14.—GEOMETRIC MEAN FAT YIELDS OF ORIGINAL-ENTRY AND REENTRY COWS

difference between the trend in the yearly fat yields of the original-entry cows (whole population) and the trend in the yearly fat yields of the reentry cows as age advances. The yearly fat yields of the reentry cows besides being far superior to the yearly fat yields of the original-entry cows, increase at a greater rate during the period of growth and decrease at a slower rate during the period of senility. This same relationship may be deduced from a comparison of the velocity constants in the equations of these smooth curves in Fig. 14. The velocity constants ($100k_1$), determining the percentage rate of decline in the growth power of the mammary gland cells, are practically the same for both groups of cows, being — 23.49 percent for the

original-entry cows and — 23.03 percent for the reentry cows. On the other hand, in the last terms, de^{kt} , which determine the rate of decline in the physiological activity of the mammary glands,^a both d and k_2 are greater for the original-entry than for the reentry cows. In other words, the more rapid rate of increase as well as the less rapid rate of decrease in the mammary activity of the reentry cows is due almost entirely to the distinctly lower rate of decline in the physiological activity of the cells composing their mammary glands.

Since the relative physiological activity of the mammary gland cells decreases at a greater rate in the original-entry than in the reentry cows, it necessarily follows that the original-entry cows should reach the stage of maximum production sooner in life than the reentry cows. Such is the case, for the age of maximum production in the original-entry cows is 7 years 4.42 months, whereas in the reentry cows it is 8 years 9.22 months. Hence it is obvious that the original-entry and reentry Jersey cows are distinctly different in their courses of development as measured by the physiological activity of their mammary glands with advancing age. It will be interesting to note at this point that Robertson and Ray (1920) have found that overgrowth in mice, due to heavy feeding, is correlated with late maturity and long life. The individuals showing overgrowth are very highly resistant to external disturbing factors and tend towards a relative paucity of tissue (inert tissue) accretion late in life. In view of these results of Robertson and Ray, the later maturity and slower rate of senescence in the reentry cows, as measured by the activity of their mammary glands, may be attributed *in part* to the more favorable environment under which the cows are kept.

Difference in Genetic Constitution for Milk Production Between Original-Entry and Reentry Cows

It was pointed out early in the discussion that there is a tendency on the part of breeders to select only the better-producing cows for reentry in the Register of Merit. This selection if practiced to an appreciable extent would bring about a genetic difference for milk production between the original-entry and reentry cows, and such seems to be the case. The present data and likewise the data of Graves and Fohrman (1925), who also made a separate study of the yearly fat yields of the original-entry and reentry Register-of-Merit Jersey cows, illustrates this genetic superiority of the reentry cows. The arithmetic mean fat yields at successive ages for reentry cows,

^aSee footnote, page 218.

and the original entries of the reentry cows, as also the original entries of cows without reentry records, are shown in graphs I, II, and III respectively in Fig. 15. It will be noted that up to the age of maturity the original-entry fat yields of the reentry cows (II) lie somewhat above and tend to parallel the yearly fat yields of the other original-entry cows (III). Beyond maturity, however, there is very little difference between the yearly fat yields of the two groups of cows. Graves assumed that the difference between the original-

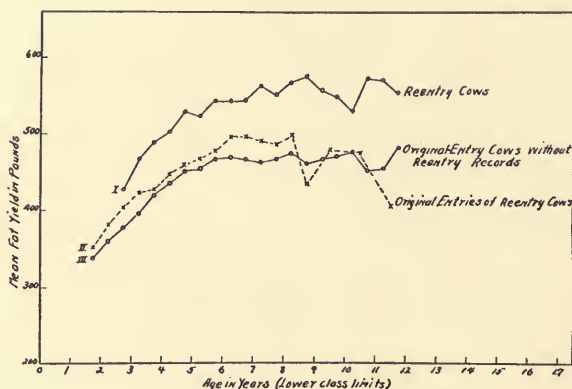


FIG. 15.—ARITHMETIC MEAN FAT YIELDS OF REGISTER-OF-MERIT JERSEY COWS
(Data from Graves, U. S. D. A. Bul. No. 1352)

entry yearly fat yields of the reentry cows (II) and their subsequent reentry fat yields (I) was due to the superior development of the cows brought about by their better environmental conditions. Such an assumption, however, is not entirely warranted as there is still an opportunity for further selection of the cows entered for the third, fourth, fifth, etc., times. Just as the cows to be entered for the second time are selected on their superior productive ability from all the original-entry cows, so may the cows entered for the third time be selected on their superior productive ability from all of the second-entry cows. In other words there may be a continual selection of the cows each time they are chosen for further entry in the Register of Merit. Such a selection, when coupled with the superior development of the cows resulting from their better environmental conditions, would naturally boost the reentry fat-yield curve higher and higher above the original-entry fat-yield curve with advancing age. Hence there is no doubt but that the superior productive ability of the reentry cows is due to the influence of both environmental and

hereditary factors, but owing to complexity of conditions it is well nigh impossible to determine the exact effect of either environment or heredity.

INFLUENCE OF LEVEL OF PRODUCTION UPON AGE CURVE OF MILK SECRETION

The percentiles of the yearly fat-yield frequency distributions for the original-entry and reentry Jersey cows are shown in Figs. 16 and 17 respectively. These percentiles were computed from the calculated (smoothed) means (a) and standard deviations (s) reported in Tables 15 and 16 respectively. In Figs. 16 and 17 the 50-percentiles are

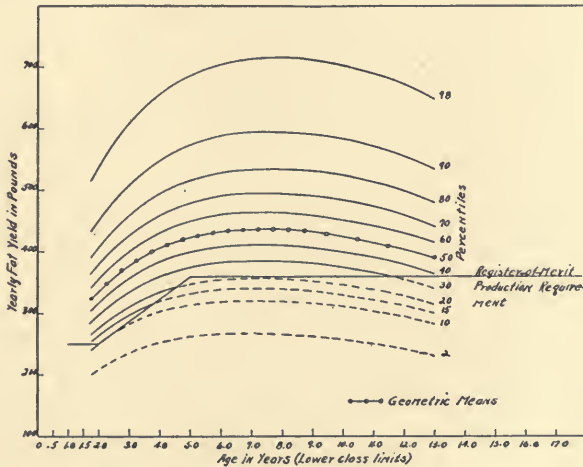


FIG. 16.—PERCENTILES OF YEARLY FAT YIELDS FOR
ORIGINAL-ENTRY JERSEY COWS

represented by the geometric mean fat yields, above and below which lies 50 percent of the total area of the yearly fat-yield frequency curves. It will be noted that a given percentage of the area of the frequency curves above their means takes in a greater range in fat yield than the same percentage of the area of the curve below their means. This difference in the range of fat yield spanned by equal percentage areas of the frequency curves above and below their means is due, of course, to the skewness of the yearly fat-yield frequency distributions, which in turn has been interpreted as being the result of the constant percentage effects of the factors determining the rate of milk secretion.

Another point of interest which may be deduced from these yearly fat-yield percentiles is the fact that at the higher levels of production there is a somewhat greater percentage increase in the milk secretion up to the age of maximum production than at the

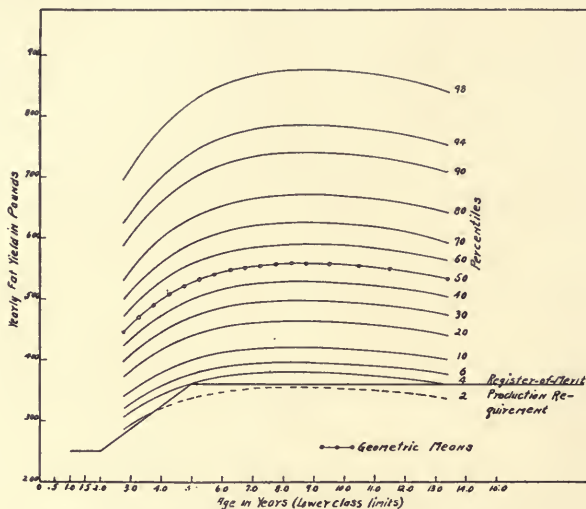


FIG. 17.—PERCENTILES OF YEARLY FAT YIELDS FOR REENTRY JERSEY COWS

lower levels of production, and a correspondingly smaller decrease thereafter. Another consequence is that at the higher levels the age of maximum production comes somewhat later in life than at the lower levels. These result from the slight increase in the values of s with age.

NATURE OF SELECTION OF REGISTER-OF-MERIT PRODUCTION REQUIREMENT

Review of Literature.—The nature of the selection of the Register-of-Merit production requirement has been a matter of much concern ever since the establishment of the Register of Merit by the American Jersey Cattle Club. Almost every investigator who has studied the production records of the Register-of-Merit cows has in some way or other referred to the selective effects of the requirement, but none have actually estimated the percentage of cows eliminated. Gowen (1921) made a study of the nature of the selection of the Register-of-Merit production requirement and concluded that the cows under 2 years of age and over 11.8 years of age were handicapped more by the requirement than the cows of the intervening ages. He also recognized that the cows in the neighborhood of 5 years of age were handicapped more than the cows of other intervening ages. Gowen, however, made no estimate of the percentage of all the cows eliminated by the requirement at the various ages. Hooper (1921) also made a study

of the nature of the selection of the Register-of-Merit production requirement and concluded that 2-year-old Jersey cows were handicapped the least and 5-year-old cows the most by the selective influence of the requirement. Here again no estimate was made concerning the percentage of all the cows eliminated at the various ages.

The area of the yearly fat-yield frequency distributions at successive ages truncated by the Register-of-Merit production requirement for both the original-entry and reentry cows is shown in Figs. 16 and 17 respectively. The percentiles indicated by the broken lines under the Register-of-Merit production-requirement curves in Figs. 16 and 17 may be assumed to represent the percentage of the cows at each age eliminated by the requirement. It will be noted, for the original-entry cows, that the production-requirement curve just includes the 10-percentile for cows 2 years of age and then increases linearly with age, cutting the 15- and 20-percentiles, up to the 23-percentile for cows 5 years of age. Above the age of 5 years the production requirement remains constant at 360 pounds, but decreases in its selective effect as it nears the age of maximum production, where it just touches the 20-percentile. Beyond the age of maximum production the requirement again begins to increase in its selective effect and continues to increase thruout the remaining life of the cows, cutting the 39-percentile at the age of 13 years. Hence it may be assumed that the percentage of all the original-entry cows at each age eliminated by the requirement increases from 10 percent of the 2-year-old cows to 23 percent of the 5-year-old cows and then decreases to 20 percent of the 7.5-year-old cows, following which it continually increases, eliminating 39 percent of the 13-year-old cows. The production-requirement curve in Fig. 17 follows a similar trend in its selective influence as age advances, but eliminates only a very small percentage of the reentry cows. Less than 2 percent of the 2-year-old reentry cows, 4 percent of the 5-year-old cows, approximately 2 percent of the 8.5-year-old cows and only 4 percent of the 13-year-old cows are eliminated by the requirement. Therefore it is obvious that the high level of production of the reentry cows almost entirely excludes them from the selective influence of the Register-of-Merit requirement.

SUMMARY

In this study the Register-of-Merit records of 9,694 original-entry and 2,628 reentry cows were analyzed separately by means of biometrical methods in an effort to determine the course of growth and senescence in Jersey cows.

Course of Growth in Body Weight.—The increase in body weight of the cows with advancing age may be expressed by the growth equation $\log_{10} W = A - be^{-kt}$ in which W is the weight at any age t , and A is the logarithm of the body weight at maturity. $100k$ is the constant percentage rate of decrease in the growth power per unit weight, e is the base of the natural logarithms, and b is a constant locating the curve in point of time. The formulas of the equations representing the growth data for the original-entry and reentry cows are respectively:

$$\log_{10} W = 2.9793 - .1273e^{-.2762t}$$

$$\log_{10} W = 2.9930 - .1344e^{-.2993t}$$

A comparison of the constants in these equations shows that there is a distinct difference between the course of growth in body weight in the original-entry and reentry cows. The reentry cows attain a greater weight at maturity and increase in weight more rapidly than do the original-entry cows. Both groups reach their maximum weight at approximately 8 years of age. It was found that there is no genetic difference between the original-entry and reentry cows for body size; hence it may be assumed that the greater size and the more rapid rate of growth of the reentry cows is due largely to the more favorable environment under which they are kept.

Course of Growth and Senescence as Described by Rise and Fall in Yearly Butterfat Yields with Advancing Age.—It was necessary first to correct for the truncation of the yearly butterfat frequency distributions of the cows, due to the selective effect of the production requirement of the Register of Merit. The frequency distributions of the reentry cows at successive ages are only slightly truncated at the lower levels, whereas the distributions of the original-entry cows are severely truncated. It was found that the frequency curve best adapted to these yearly fat-yield frequency distributions of both the original-entry and reentry cows is the log-transformed equation of the normal curve

$$y = \frac{1}{sx\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\log x - a}{s}\right]^2}$$

in which a and s are the mean and standard deviation respectively on the log scale. Owing to the severe truncation of the original-entry distributions, it was necessary to fit them in a peculiar manner in order to determine the curves of the whole populations of which the original-entry cows are a truncated sample. The percentage of the cows eliminated by the production requirement was estimated from the fitted frequency curves. Only 2 percent to 4 percent of the re-entry cows are eliminated, whereas 10 percent to 39 percent of the original-entry cows are eliminated.

The true means of the yearly fat yields at successive ages for both the original-entry and reentry cows were determined by the fitted log-transformed frequency curves. These means increase with advancing age up to a maximum (age of maximum production) but at an ever-decreasing rate, and then decrease at an ever-increasing rate. This rise and fall in the yearly fat yields of the cows may be expressed in part by an equation of the same type as the growth equation representing the increase in body weight with age. However, a corrective factor must be added to this equation in order to take into account the decrease in the fat yields after the age of maximum production. The corrected equation takes the form of $\log_{10} M = A - be^{-k_1t} - de^{k_2t}$ in which M = fat yield at any age t . The first part of this equation, $\log_{10} M = A - be^{-k_1t}$ broadly speaking, may be interpreted as representing the increase in the fat yield with advancing age due to the growth of the mammary gland, $100k_1$ being the constant percentage rate of decrease in the growth power per unit volume of the gland. The total function of the mammary gland, however, depends not only upon the number of cells composing it, but also upon the relative physiological activity of the cells. The physiological activity of the cells depends upon the amount of active protoplasm within them, and since this constantly decreases with advancing age and apparently at an ever-increasing rate, their relative physiological activity likewise decreases. Hence the corrective term de^{k_2t} may be said to represent the decline in the relative physiological activity of the cells in the mammary gland accompanying growth and senility.

The theory of senescence involved in the above interpretation may be said to be in accordance with the general theory of senescence advanced by Child (1915). Consequently the process of senescence is indicated just as clearly in the ever-decreasing rate of increase in the mean fat yields up to the age of maximum production as in the ever-increasing rate of decrease in the mean fat yields beyond the age of maximum production.

The formulas of the equations representing the mean fat yields with advancing age for the original-entry and reentry cows are respectively:

$$\log_{10} M = 2.6757 - .2038e^{-.2349t} - .00596e^{.1133t}$$

$$\log_{10} M = 2.7733 - .2366e^{-.2303t} - .00456e^{.0953t}$$

These equations when graphed show that there is a distinct difference between the rise and fall in the yearly fat yields of the original-entry and the reentry cows. The fat yields of the reentry cows are far superior to the fat yields of the original-entry cows and increase at a greater rate with advancing age. After the age of maximum production (during senility) the fat yields of the reentry cows do not decline so rapidly as do the fat yields of the original-entry cows. The age of maximum production in the reentry cows is 8 years 9.22 months and in the original-entry cows 7 years 4.42 months.

It was found that the genetic difference for milk production between the original-entry and reentry cows, altho significant, was not great enough to account entirely for the superior productive ability of the latter. Hence it may be assumed that both heredity and environment play an important part in bringing about the distinctly superior rate of milk secretion in the reentry cows.

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APPENDIX

Fitting Log-Transformed Normal Frequency Curve by Method of Least Squares to Truncated Yearly Butterfat Frequency Distributions of Original-Entry Register-of-Merit Jersey Cows

In fitting frequency curves Pearson (1900) has shown that $\chi^2 = \frac{(f_o - f_c)^2}{f_o}$, where f_o = observed frequency and f_c = calculated frequency, should be minimum. Thus an ordinary least-square fit, $(f_o - f_c)^2 = \text{minimum}$, gives too much weight to the high values of f at the expense of the low values. The proper weighting may be secured by multiplying the squared residuals by $\frac{1}{f_c}$. For example, the following differences contribute equally to χ^2 :

f_o	f_c	Δf	$(\Delta f)^2$	$\frac{1}{f_c} (\Delta f)^2$
1.10	1	.10	.01	.01
10.31	10	.31	.10	.01
101.00	100	1.00	1.00	.01

In actual calculation, however, $\frac{1}{f_o}$ must be used as an approximation to $\frac{1}{f_c}$. This, of course, introduces an error, but one of no practical importance.

In fitting the log of frequency curves, a residual Δf in the primary curve is represented by $\frac{\Delta f}{f}$ in the log curve (since $d \log f = \frac{df}{f}$) and the squared residual by $\frac{(\Delta f)^2}{f^2}$. Therefore in fitting the log frequencies by least squares, each squared residual should be weighted by f in order to make $\frac{(\Delta f)^2}{f}$ minimum, since $\frac{(\Delta f)^2}{f^2} \cdot f = \frac{(\Delta f)^2}{f}$. For example,

f_o	f_c	$\log f_o$	$\log f_c$	$\Delta \log f$	$(\Delta \log f)^2$	$f_c (\Delta \log f)^2$
1.10	1	.041	0	.041	.0016	.0016
10.31	10	1.013	1	.013	.00016	.0016
101.00	100	2.004	2	.004	.000016	.0016

Here again f_o instead of f_c must be used in weighting in the actual calculation.

The equation of the log-transformed normal curve fitted to the truncated yearly fat-yield frequency distributions of the original-entry cows is

$$y = \frac{N'}{sx\sqrt{2\pi}} e^{-\frac{1}{2} \left[\frac{\log x - a}{s} \right]^2}$$

in which N' = the total frequency under the curve, including the unknown portion below the Register-of-Merit production requirement; a and s are the mean and the standard deviation on the log scale. It is convenient in fitting to transform to $\log_{10} x$ instead of $\log_e x$ and to measure x in 100-pound units. As the class ranges are in 50-pound units (except for the first class) the observed frequencies

give an approximation to $\frac{1}{2}y, \frac{\text{range}}{100}y$, in general. Letting $y = f_o = \frac{y}{2}$ for complete classes

$$y = \frac{N' \log_{10} e}{2sx\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\log_{10} x - a}{s}\right]^2}$$

This equation cannot be fitted by direct methods because of the truncation of the data. It can be fitted, however, by taking the logarithms of the frequencies, which throws it into the form of a parabola, a convenient form for fitting by least squares.

Letting $y' = \log_{10} y_o$ and $x' = \log_{10} x$

$$y' = \log_{10} \left[\frac{N' \log_{10} e}{2s\sqrt{2\pi}} \right] - \frac{a^2 \log_{10} e}{2s^2} + \left[\frac{a \log_{10} e}{s^2} - 1 \right] x' - \frac{\log_{10} e}{2s^2} (x')^2$$

This is of the general form $y' = A + Bx' + C(x')^2$. Where

$$A = \log_{10} \left[\frac{N' \log_{10} e}{2s\sqrt{2\pi}} \right] - \frac{a^2 \log_{10} e}{2s^2}$$

$$B = \frac{a \log_{10} e}{s^2} - 1$$

$$C = -\frac{\log_{10} e}{2s^2}$$

The constants A , B , and C were determined by the method of least squares as follows:

The general least-square normal equations of the parabola $y' = A + Bx' + C(x')^2$, using the weight f_o , are

$$\begin{array}{rcl} A & B & C \\ f_o & + f_o x' & + f_o (x')^2 = f_o y \\ f_o(x') & + f_o(x')^2 & + f_o(x')^3 = f_o x' y \\ f_o(x')^2 & + f_o(x')^3 & + f_o(x')^4 = f_o(x')^2 y' \end{array}$$

In these formulae $y' = \log_{10} f_o$ except in the first class of each distribution, which is of varying range. The effect of the truncation is illustrated in Fig. 18. Taking, for example, cows 3.5 to 4.0 years of age, the first recorded butterfat class is that for 300 to 350 pounds. This class is truncated by the requirement to varying extents indicated by the shading. An average of the truncation is at 314.4 pounds, leaving a class range of 35.6 pounds instead of the usual 50 pounds, and a class mid-point of 332.2 pounds. The observed frequency of 104 must be rated up by the ratio $\frac{50}{35.6}$ (giving 146.1) to obtain the magnitude of the ordinate on a scale comparable to those for the complete classes. In this case then $y' = \log_{10} \left(\frac{50}{\text{range}} \right) f_o$.

The whole procedure for fitting this age class is given in Table 17.

The method of determining the theoretical frequencies (f) is reported in Table 18 and is the same as that described by Wright (1926). N'' in Table 18

TABLE 17.—PROCEDURE FOLLOWED IN FITTING LOG-TRANSFORMED FREQUENCY CURVE BY METHOD OF LEAST SQUARES TO FAT-YIELD FREQUENCY DISTRIBUTIONS OF 3.5- TO 4.0-YEAR-OLD ORIGINAL-ENTRY COWS

Fat yield class mid-point x	f_x	x'	$f_x x'$	$f_x(x')^2$	$f_x(x')^3$	$f_x(x')^4$	u_x	y'	$f_x y'$	$f_x x' y'$	$f_x(x')^2 y'$
332.2	104	.521400	54.2256	28.2732	14.7417	7.6863	146.1	2.164650	225.1236	117.3796	61.2017
375	183	.574031	10.5047	60.3006	34.0144	19.8697	183	2.262451	414.0285	237.6652	136.4272
425	143	.628389	89.8696	56.4668	35.4832	22.2973	143	2.155336	308.2131	193.6777	121.7049
475	92	.676694	62.2559	42.1282	28.9079	19.2911	92	1.963788	180.6685	122.2573	82.7308
525	50	.720159	36.0080	25.9315	18.6748	13.4488	50	1.698970	84.9485	61.1764	44.0507
575	29	.759668	22.0304	16.7358	12.7136	9.6582	29	1.462398	42.4095	32.2171	24.4743
625	19	.795880	15.1217	12.0351	9.5785	7.6233	19	1.278754	24.2963	19.3369	15.3899
675	11	.829304	9.1223	7.5652	6.2739	5.2030	11	1.041393	11.4553	9.4999	7.8783
725	3	.860338	2.5810	2.2205	1.9104	1.6436	3.	1.477121	1.4314	1.2315	1.0595
775	2	.889302	1.7786	1.5817	1.4066	1.2509	2	.301030	.6021	.5555	.4762
Total.....	636	398.0308	253.2386	163.9051	107.9722	1293.1768	794.9770	495.3995

Least square equations:

$$\begin{aligned}
 A &+ 398.0308 + 253.2386 = 1293.1768 \\
 B &+ 253.2386 + 163.9051 = 794.9770 \\
 C &+ 163.9051 + 107.9722 = 495.3995 \\
 A &= -3.551911 \\
 B &= +20.624279 \\
 C &= -18.389400
 \end{aligned}$$

$$\begin{aligned}
 C &= -18.389400 = -\frac{\log_{10} e}{2a^2}, a = .10866 \\
 B &= 20.624279 = \frac{a \log_{10} e}{s^2} - 1, a = .58795 \\
 A &= -3.551911 = \log_{10} \left[\frac{N' \log_{10} e}{2^n \sqrt{2\pi}} \right] - \frac{a^2 \log_{10} e}{2s^2}, N' = 800.8
 \end{aligned}$$

TABLE 18.—PROCEDURE FOLLOWED IN CALCULATING THEORETICAL FREQUENCIES FOR ORIGINAL-ENTRY FAT-YIELD FREQUENCY DISTRIBUTIONS REPORTED IN TABLE 17

Fat yield pounds 100z	x'	$x' - a$	$\frac{x' - a}{s}$	$\text{Prf} \left(\frac{x' - a}{s} \right)$	Percent of f_c	f_c	f_o	$(f_o - f_c)$	$\frac{(f_o - f_c)^2}{f_o}$
100-150	.17609	-.41186	-3.790	-49.992	.008	.06
-200	.30103	-.28692	-2.642	-49.586	.406	3.24
-250	.39794	-.19001	-1.749	-45.985	3.601	28.71
-300	.47712	-.11083	-1.020	-34.613	11.372	90.68
-314.4	.49748	-.09047	-.833	-29.757	14.068	112.18	104	- 8.18	.5965
-350	.54407	-.04388	-.404	-15.689	18.924	150.90	183	+16.65	1.6665
-400	.60206	+ .01411	.130	+ 5.172	20.861	166.35	143	+ 3.96	.1128
-450	.65321	+ .06526	.601	+22.608	17.436	139.04	192	+ 4.11	.1758
-500	.69897	+ .11102	+1.022	+34.661	12.053	96.11	50	- 8.37	1.1737
-550	.74036	+ .15241	+1.403	+41.969	7.308	58.27	29	- 3.10	.2994
-600	.77815	+ .19020	+1.750	+45.994	4.025	32.10	50	+ 3.10	.3439
-650	.81291	+ .22496	+2.070	+48.077	2.083	16.61	19	+ 2.82	.9722
-700	.84510	+ .25715	+2.367	+49.588	1.485	8.18	3	+ 2.15	.6465
-750	.87506	+ .28711	+2.642	+49.813	.225	3.87	2
-800	.90309	+ .31514	+2.900	+49.916	.103	1.79
-850	.92942	+ .34147	+3.143084	.67
-900
Total.....	100.000	797.4	636	$\chi^2 = 5.9873$

$N'' = 797.4, n = 6, n' = 7, P = .43.$

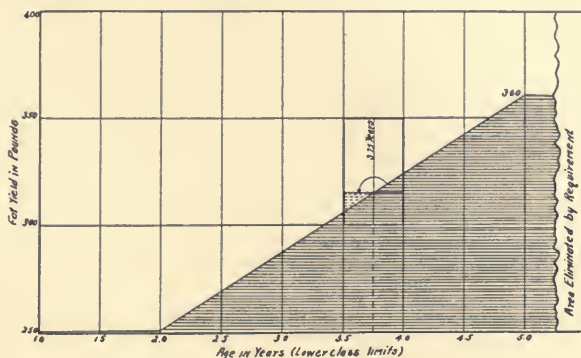


FIG. 18.—REGISTER-OF-MERIT PRODUCTION REQUIREMENT CURVE

was calculated from the total frequency of the truncated data, $\sum f_0 = 636$. As 79.8 percent of the area of the theoretical curve is above the point of truncation (314.4 pounds) and 636 cows are recorded above this point, the percentage frequencies must be multiplied by $\frac{636}{79.8} = 7.974$ to obtain the theoretical frequencies which will give minimum χ^2 . This gives a total theoretical frequency, $N'' = 797.4$, slightly different from the figure $N' = 800.8$, obtained from the solution of the normal equations. The latter is used only as a rough check to N'' . In calculating the probability from χ^2 , it must be noted that 3 degrees of freedom are lost in the calculation of N' , a , and s from the data (see Fisher 1925). The 9 contributions to χ^2 thus yield 6 degrees of freedom and in Elderton's table are entered under $n' = 7$ (n' being one greater than the number of degrees of freedom).





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